Optimization in modern power systems

Lecture 13: Final Review

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Review of Week 1

By the end of Week 1 you must be able to:

1. Formulate an optimization from a given problem description and solve it
2. Formulate the Economic Dispatch problem and solve it
3. Formulate the DC-OPF problem and solve it
4. Formulate the Bus Susceptance Matrix, and formulate and explain the PTDFs
5. Draw the Merit Order curve, and identify the marginal generators
6. Formulate the Lagrangian of a constrained optimization problem
7. Derive the KKT conditions
8. Derive the LMPs from the DC-OPF problem

Some take-aways from Week 1

- (The list is not complete!)
- The AC-OPF is a non-linear non-convex problem
- The DC-OPF is a linear convex problem
  - Note: The DC-OPF can also be formulated as a quadratic problem, if instead of a linear objective function, we include quadratic costs in the objective function. This is still convex.
- The Economic Dispatch assumes a copperplate network: no consideration of power flows, assuming that there are no congestions.
  - The Economic Dispatch is also a convex problem.

Some take-aways from Week 1 (cont.)

- Locational Marginal prices in *DC-OPF*
  - LMPs are equal on all nodes if there is no congestion
  - LMPs are different on different nodes if there is congestion
- Calculate the LMPs
  - Standard Power Flow equations: the lagrangian multipliers ν_i of the equality constraints
  - PTDFs: $\text{LMP}_i = -\nu - \sum_{lm \in L} PTDF_{lm,i} \cdot \lambda_{lm}$, where ν is lagrangian multiplier of the equality constraint, and λ_{lm} is the Lagrangian multiplier of the congested line $l - m$. If line $l - m$ is not congested, then $\lambda_{lm} = 0$

Review of Week 2

By the end of Week 2 you must be able to:

1. formulate an AC-OPF problem
2. formulate the Bus Admittance Matrix and the Line Admittance Matrix of both the DC-OPF and the AC-OPF problems
3. list the three simplifications that get us from the AC power flow equations to the DC power flow equations
4. understand why the formulation of the dual problem is important
5. describe strong duality, weak duality, and duality gap
6. list the main difference between the SDP and the LP problem
7. name at least one condition to determine a positive semidefinite matrix
8. describe the reformulation of quadratic constraints to an SDP problem and the associated convex relaxations
9. describe when such a convex relaxation is tight

Some take-aways from Week 2

- (The list is not complete!)
- $S_G - S_L = diag(\overline{V})\overline{Y}^*_{\text{bus}}\overline{V}^*$
- Line flow $i \rightarrow j$ is different than $j \rightarrow i$. We need to consider both directions
- AC$\rightarrow$ DC: $Z_{ij} = jX_{ij},\ V = 1pu,\ \sin\theta \approx \theta$
- dual: concave, lower bound, might be easier to solve; if strong duality $\rightarrow$ exact!
- P positive semidefinite $\rightarrow$ P must be symmetric and: all its eigenvalues are non-negative or all its principal minors non-negative[1]
- quadratic constraints to SDP: $Y = xx^T \Rightarrow Y \succeq 0$ and rank$(Y) = 1$
- convex relaxation: drop the rank-1 constraint

[1] There are also other equivalent conditions not listed here.

AC-OPF[2]

obj.function	$\min c^T P_G$
AC flow	$S_G - S_L = diag(\overline{V})\overline{Y}^*_{\text{bus}}\overline{V}^*$
Line Current	$\lvert\overline{Y}_{\text{line},i\rightarrow j}\overline{V}\rvert \leq I_{line,max}$
	$\lvert\overline{Y}_{\text{line},j\rightarrow i}\overline{V}\rvert \leq I_{line,max}$
or Apparent Flow	$\lvert\overline{V}_i\overline{Y}^*_{\text{line},i\rightarrow j,\text{i-row}}\overline{V}^*\rvert \leq S_{i\rightarrow j,max}$
	$\lvert\overline{V}_j\overline{Y}^*_{\text{line},j\rightarrow i,\text{j-row}}\overline{V}^*\rvert \leq S_{j\rightarrow i,max}$
Gen. Active Power	$0 \leq P_G \leq P_{G,max}$
Gen. Reactive Power	$-Q_{G,max} \leq Q_G \leq Q_{G,max}$
Voltage Magnituge	$V_{min} \leq V \leq V_{max}$
Voltage Magnituge	$V_{min} \leq V \leq V_{max}$
Voltage Angle	$\theta_{min} \leq \theta \leq \theta_{max}$

[2] All shown variables are vectors or matrices. The bar above a variable denotes complex numbers. $(\cdot)^*$ denotes the complex conjugate. To simplify notation, the bar denoting a complex number is dropped in the following slides. Attention! The *current* flow constraints are defined as *vectors*, i.e. for all lines. The apparent power *line* constraints are defined *per line*.

Some Questions

1. What are the differences in the formulation of the DC-OPF based on the standard power flow equations and based on the PTDFs?
   - What is the meaning of PTDFs?
   - How do we calculate the PTDFs?
2. How do we determine the minimum of a function in unconstrained optimization?
3. What is a convex, a concave, and a non-convex function? Give an example for each.
4. Are there guarantees that we can find the global optimum of a convex, a concave, or a non-convex function? Explain for each case.
5. What is the sign of the Lagrangian multipliers for the equality and inequality constraints. Explain.
6. Draw a graphical solution of 2-dimensional optimization problem.

Some Questions (cont)

7. What are the differences between the AC-OPF and the DC-OPF?
   - Think about: type of problem, optimization variables, constraints, possible objective functions
8. How do we define a bus admittance matrix?
9. How do we define a line admittance matrix?
10. What are the assumptions we make to simplify the AC power flow equations to the DC power flow equations?
11. Multi-objective optimization: what is the role of the weights and about what must we be careful?
12. What are the reasons that we might prefer to solve the dual problem?

Some Questions (cont.)

13. Explain why the dual problem will always give a lower bound to the objective value of the primal problem
14. Explain what is strong duality, weak duality, and duality gap. How can the duality gap be used in an optimization solver?
15. Formulate the KKT conditions for an arbitrary optimization problem
16. Is a quadratic program always convex? If not, what is the condition for it to be convex? Is a QP DC-OPF that minimizes generation costs convex?
17. Formulate explicitly the objective function and the constraints for an LP DC-OPF problem
18. What is the reason for formulating the AC-OPF problem as an SDP?
19. What is the difference between SDP and LP?
20. Name one condition that determines if a matrix is positive semidefinite?

Some Questions (cont.)

21. What is the reformulation and the relaxation we do in the MAX-CUT problem? (note: a very similar relaxation takes place in the SDP formulation of an AC-OPF)

22. When is the convex relaxation tight? i.e. when does the result of our SDP problem correspong to the global optimum of our original problem?

Points you would like to discuss?