

# HVDC Line Placement for Maximizing Social Welfare – An Analytical Approach

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# Motivation

- Transmission expansion planning is a complex problem
- Take advantage of the power system properties, in order to derive simple rules which can be generally applied
- Use these results either directly, or within optimization procedures in order to bound the search-space
- Focus is on placement of HVDC lines



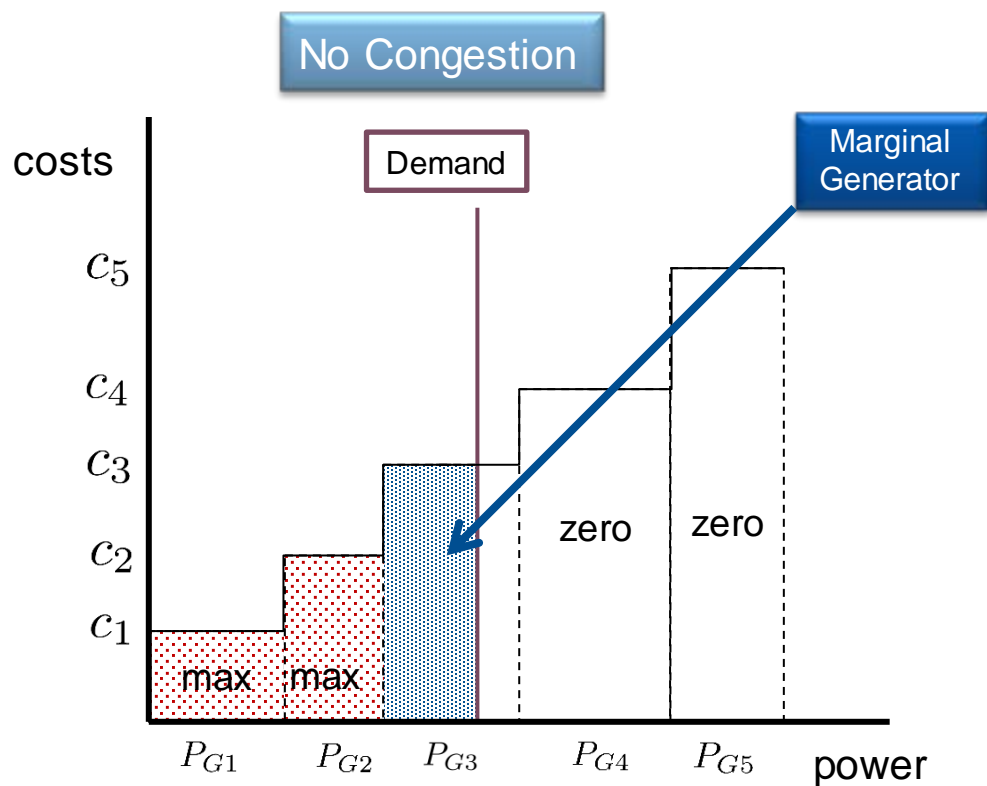
# OUTLINE

1. Overview of the Method
2. Case Study: AC-OPF on a 10-bus network
3. Description of the Algorithm
4. Case Study: European Network
5. Conclusions

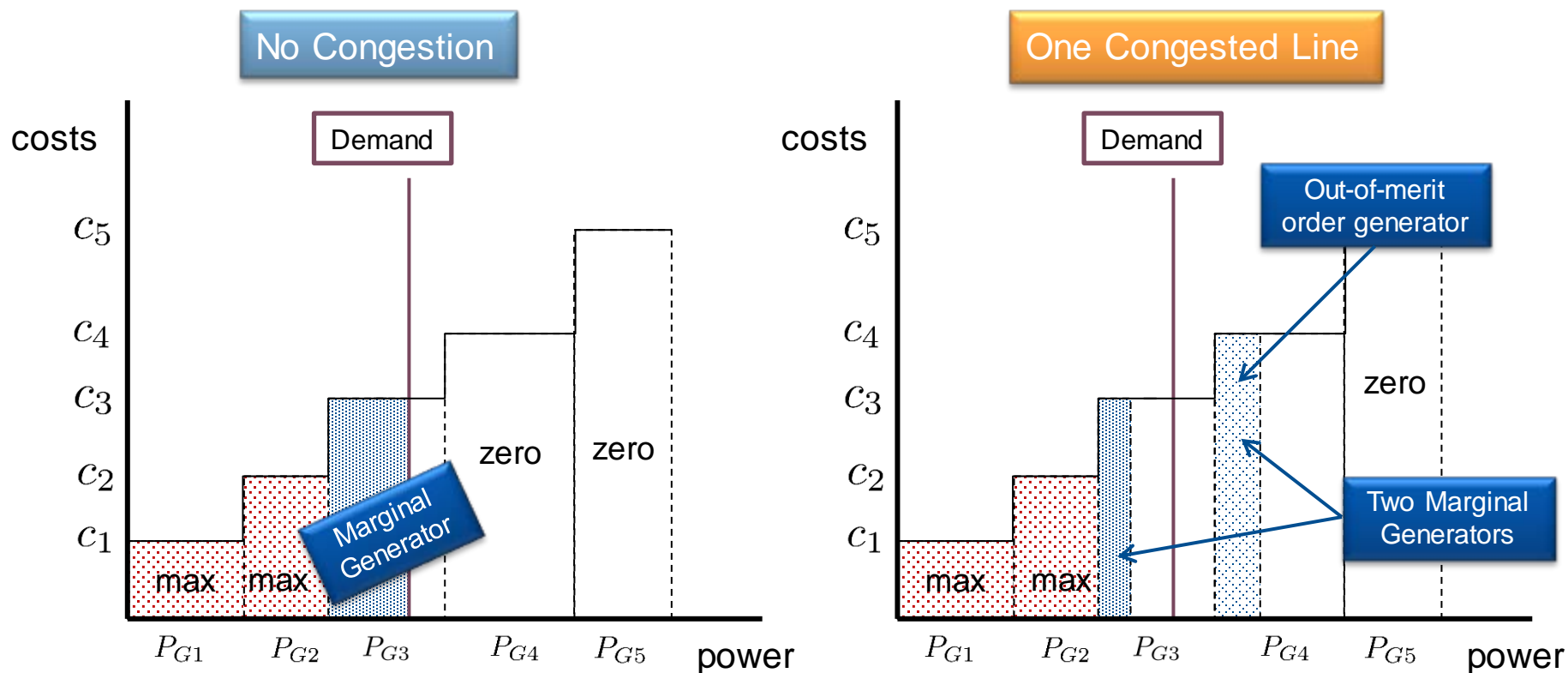
# Main Results

- In order to maximize social welfare the HVDC lines should connect a high-cost with a low-cost marginal\* generator
- *\*Marginal* is the generator which is dispatched *neither at its maximum nor at its minimum limit*
- *Upper bound*: any line with installation costs higher than the line connecting two marginal generators should be discarded
- Algorithm to identify optimal HVDC placement and estimate cost savings

# Merit-order curves with and without congestion



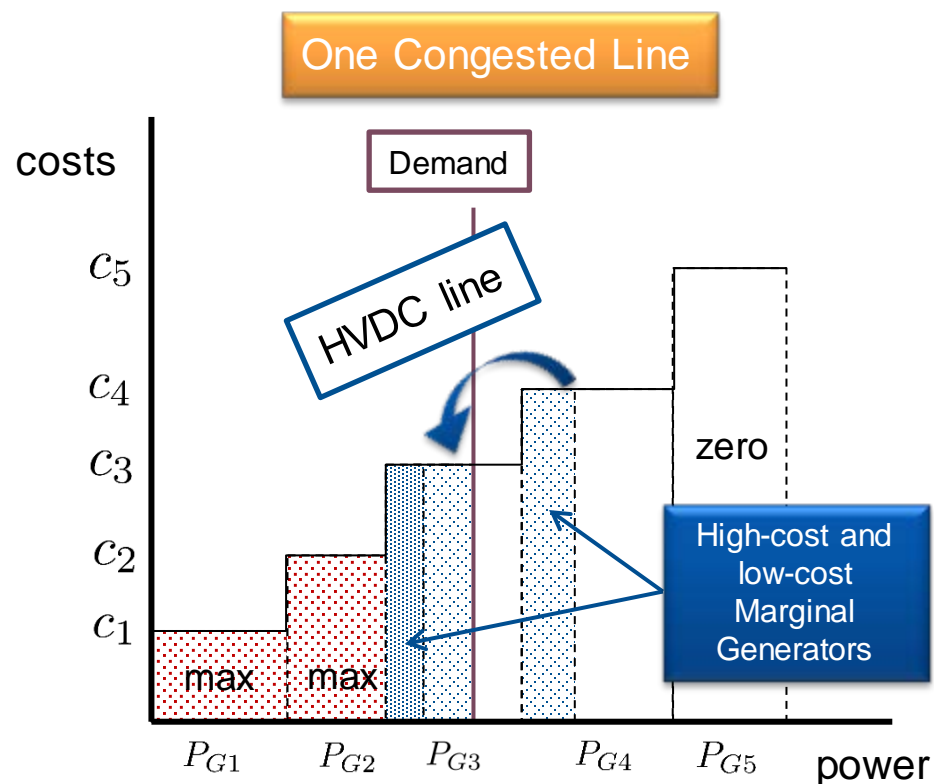
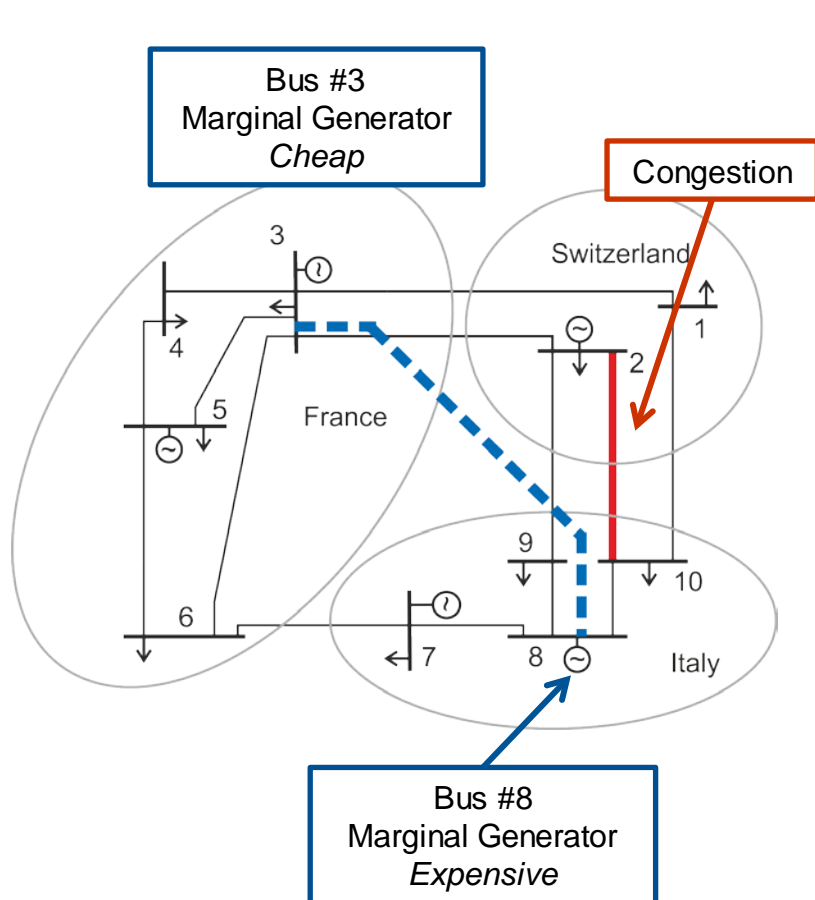
# Merit-order curves with and without congestion



- No congestion = 1 marginal generator
- 1 congested line = 2 marginal generators

# HVDC placement and marginal generators

- “Shifting” power from the high-cost to the low-cost marginal generator



## Main Results

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## Derivation of the results – Outline

- Congestion relief is equivalent to maximization of the social welfare



- Eliminating the dispatch of the high-cost marginal generator is equivalent to congestion relief (analytically shown)  
—→ shift power from expensive to cheap marginal generators



- In general, for  $N$  congestions there exist  $N+1$  marginal generators (analytically shown) —→ #marg.gens bounded



- Method applicable to both AC and HVDC lines. Due to HVDC controllability, HVDC capacity can be determined independent of network topology.

$$\min \sum_{i=1}^{N_{PG}} c_i P_{G,i}$$

DC-OPF Formulation based on PTDFs

$$|P_G - P_L| \leq F_L$$

$$0 \leq P_G \leq P_{G,max}$$



$$\mathcal{L}(P_G, \nu, \lambda, \mu) = \sum_{i=1}^{N_{PG}} c_i P_{G,i} + \nu \cdot \left( \sum_{i=1}^{N_{PG}} P_{G,i} - \sum_{i=1}^{N_{PL}} P_{L,i} \right) + \sum_{k=1}^{N_L} \lambda_k (P_G - P_L)_k + \sum_{i=1}^{N_{PG}} (\mu_i^+ \cdot (P_{G,i} - P_{G,i,max}) + \mu_i^- \cdot (-P_{G,i}))$$

Lagrangian Function of the Problem

due to KKT conditions

$$\frac{\partial \mathcal{L}}{\partial P_G} = 0$$



$$c_m + \mu_k = 0$$

LM of equality constraint

$$c_n + \lambda_k = 0$$

LM of congested line

Linear Algebra manipulations of the equations resulting from the KKT optimality conditions

# Assumptions

- DC-OPF context
- Linear generator costs

# Main Findings

- For  $N$  congestions there exist  $N+1$  marginal generators
- For a given network topology, lagrangian multiplier of the the line (Line LM) dependent only on the costs of marginal generators
  - Line LM is a “marginal” measure of how a congestion relief can affect social welfare

# OUTLINE

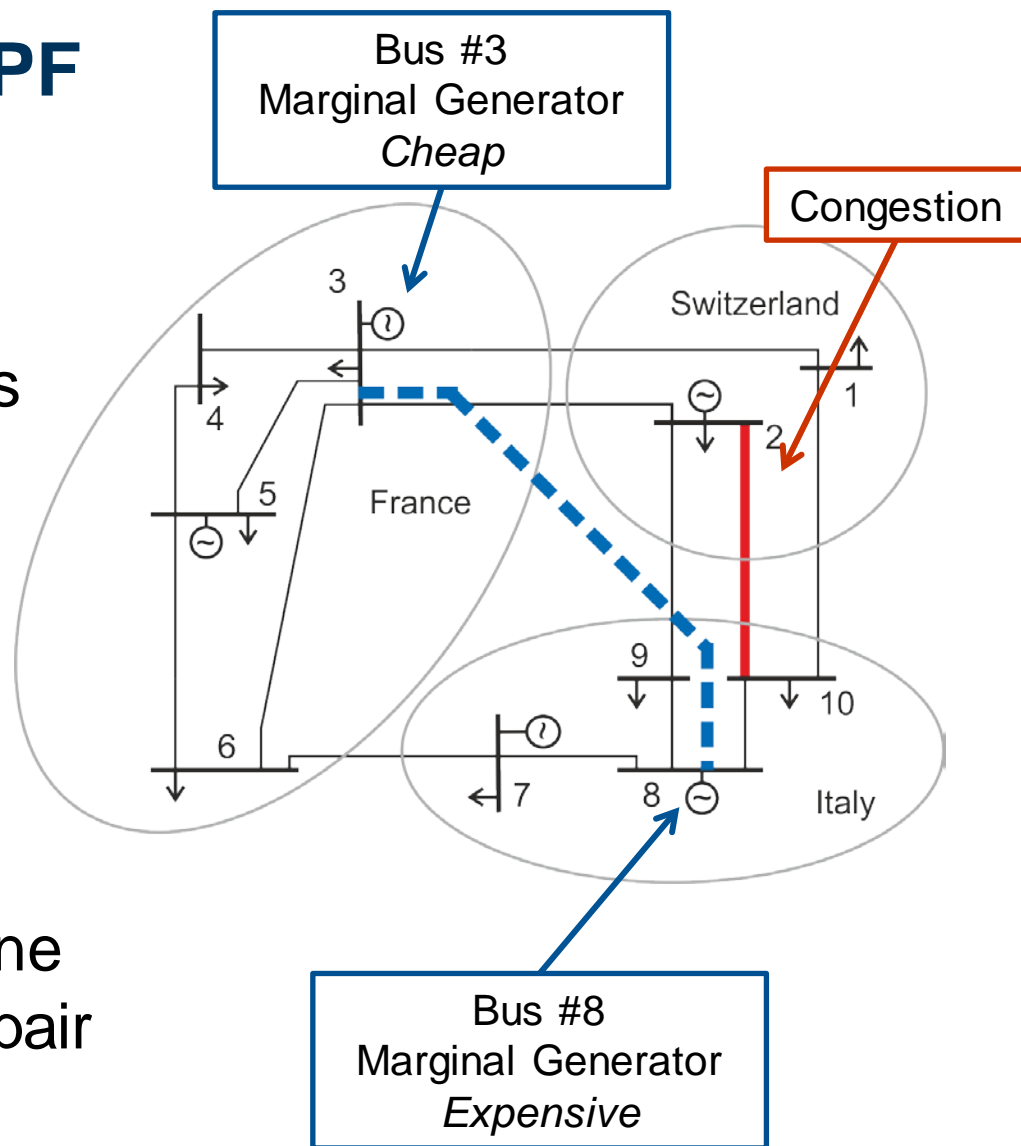
1. Derivation of the Method
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# Case Study #1: AC-OPF

- One congested line
- Two marginal generators

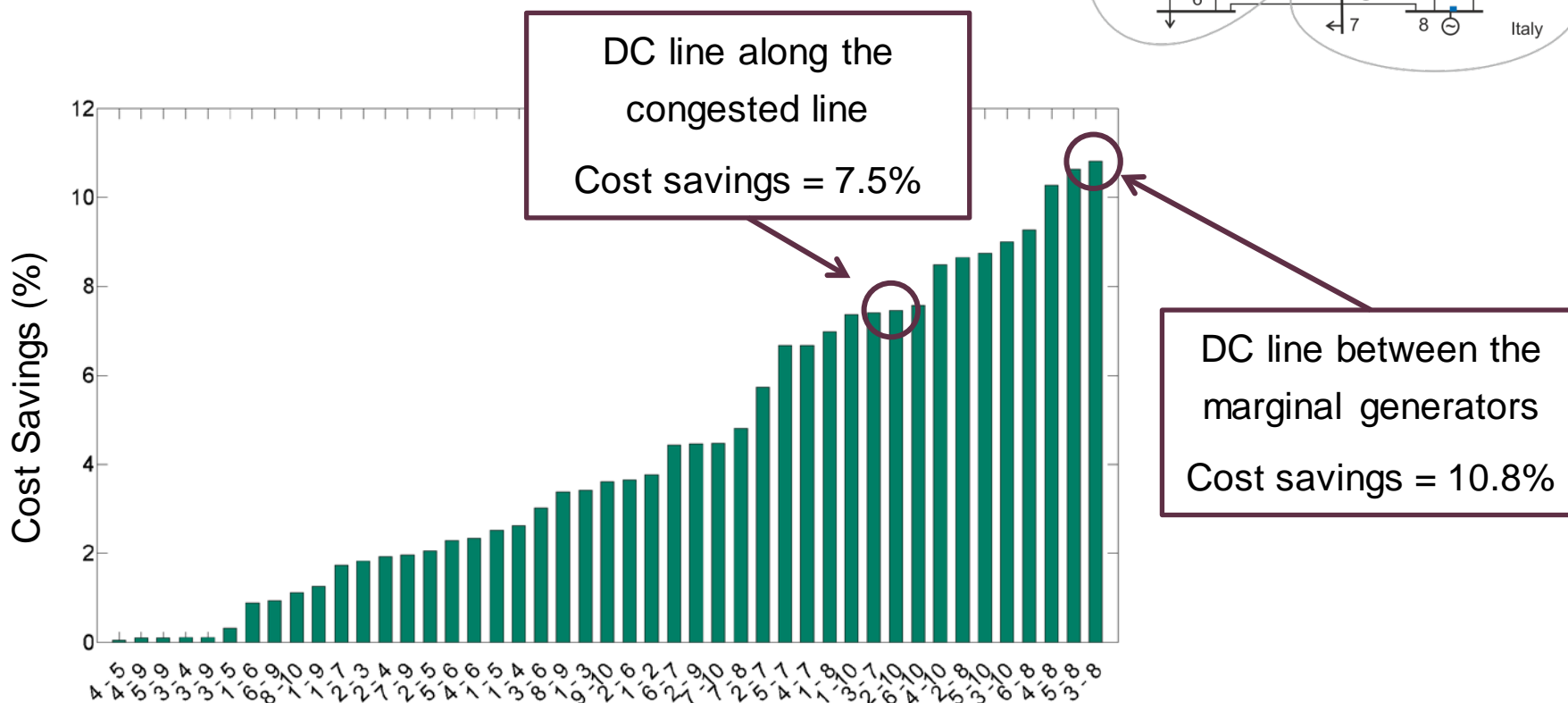
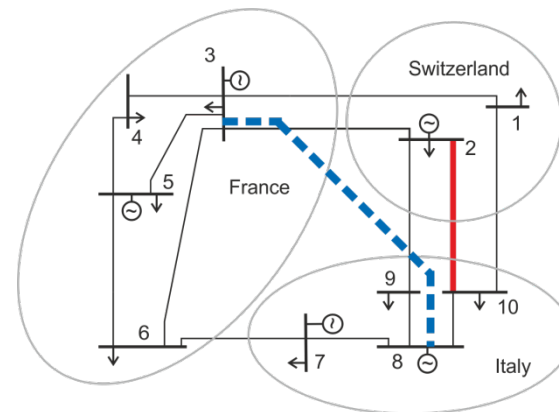
	<b>P<sub>g</sub></b> <b>(MW)</b>	<b>costs</b> <b>(€/MWh)</b>
Bus 3	6'130	24.3
Bus 8	1'530	50.0

- One 2'000 MW HVDC line between each possible pair of nodes



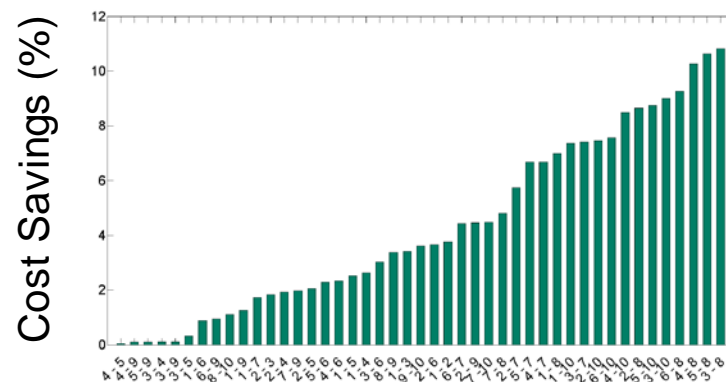
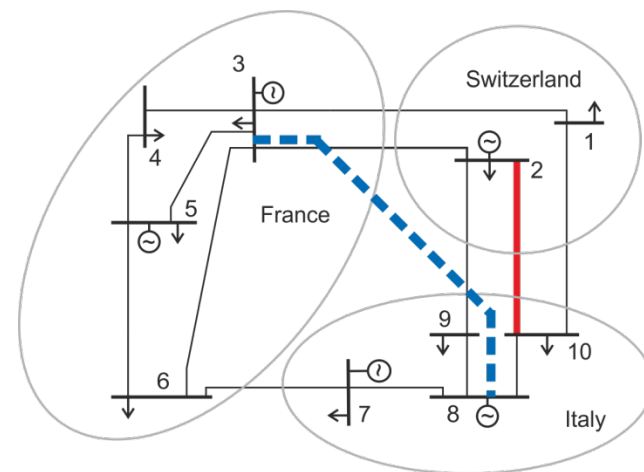
# Case Study #1: AC-OPF

- Run 45 instances of OPF for all possible placements



# Case Study #1: AC-OPF Conclusions

- DC line between marginal generators results to highest cost savings
- Placing a DC line in parallel with the congested line is not necessarily optimal
- Placement method can also hold in more realistic system representations

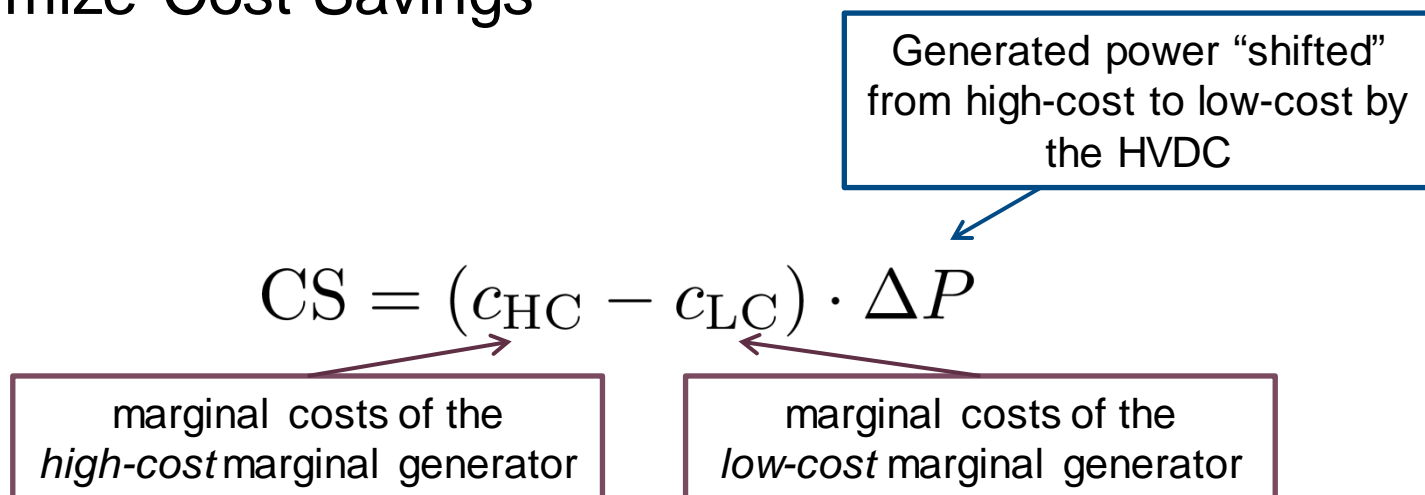


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# Algorithm – Main Idea

- Maximize Cost Savings



# Algorithm

1. Run DC-OPF and find marginal generators
2. Select HVDC capacity  $C_{DC}$
3. Find the *high-cost* marginal generator

$$G_{HC} : \max_i \{ c_{Gi} \cdot \min \{ C_{DC}, P_{Gi} \} \}$$

power currently produced

remaining unused capacity

4. Find the *low-cost* marginal generator

$$G_{LC} : \min_i \{ c_{Gi} \cdot \min \{ C_{DC}, (P_{Gi,max} - P_{Gi}) \} \}$$

5. Connect the two generator nodes with an HVDC line
6. Calculate an estimate for the cost savings

$$CS \geq (c_{HC} - c_{LC}) \cdot \min \{ C_{DC}, P_{G,HC}, (P_{G,LC,max} - P_{G,LC}) \}$$

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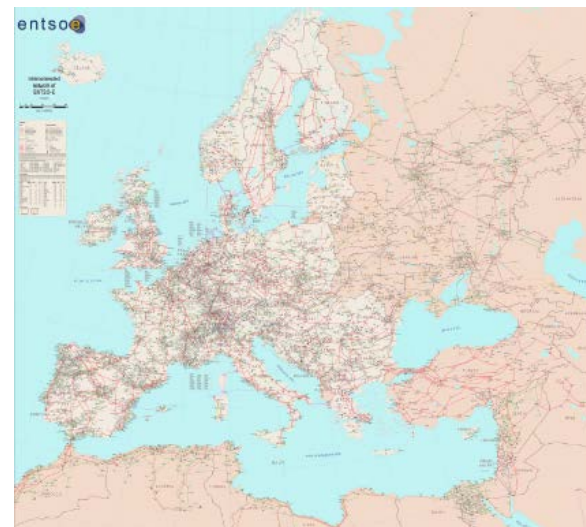
# IRENE-40 European Project

- Infrastructure Roadmap for the Energy Networks in Europe for the next **40** years



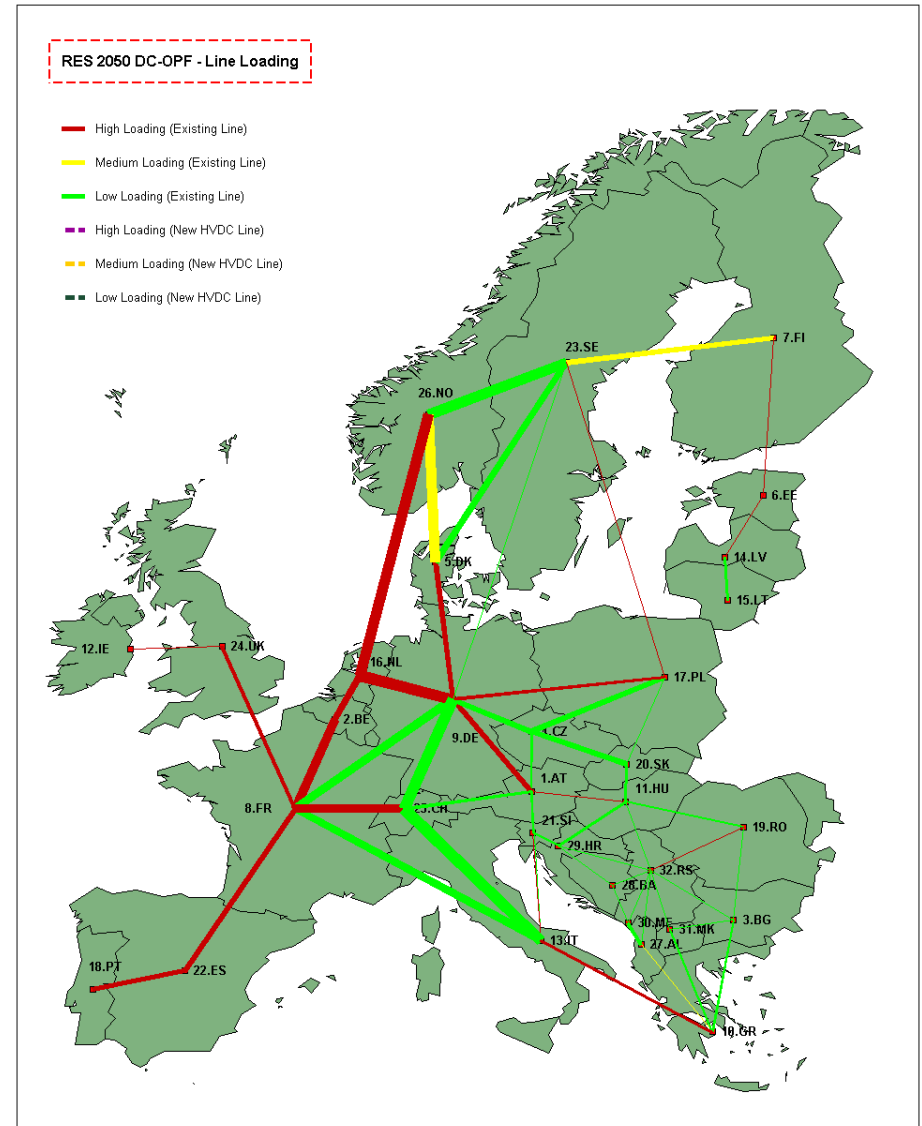
[www.irene-40.eu](http://www.irene-40.eu)

- Identify expansion measures with respect to:
  - Sustainability → more RES
  - Security
  - Competitiveness → efficient market operation



## Case Study #2: European Network

- Single-node per country;  
real network data (UCTE)
- Generation Scenario:  
80% RES by 2050
- ~70 gens: RES,  
conventional, nuclear
- Placement of 3 HVDC  
connections



# OPF Results – Marginal Generators

	P <sub>g</sub> (MW)	P <sub>g,max</sub> (MW)	Costs (€/MWh)
...	...	...	...
Ireland	7'230	10'730	0.01
Portugal	16'210	30'640	0.01
Romania	9'040	10'840	0.01
Norway	33'720	44'060	0.01
France	52'940	54'280	0.54
Sweden	1'530	11'450	0.55
...	...	...	...
Hungary	830	9'300	129.05
Poland	20'830	52'760	142.51
Spain	20'660	42'410	151.00
Greece	4'630	9'340	151.07

- Placement of 10'000 MW HVDC

$$\min\{ c_{LC} \cdot \Delta P \}$$

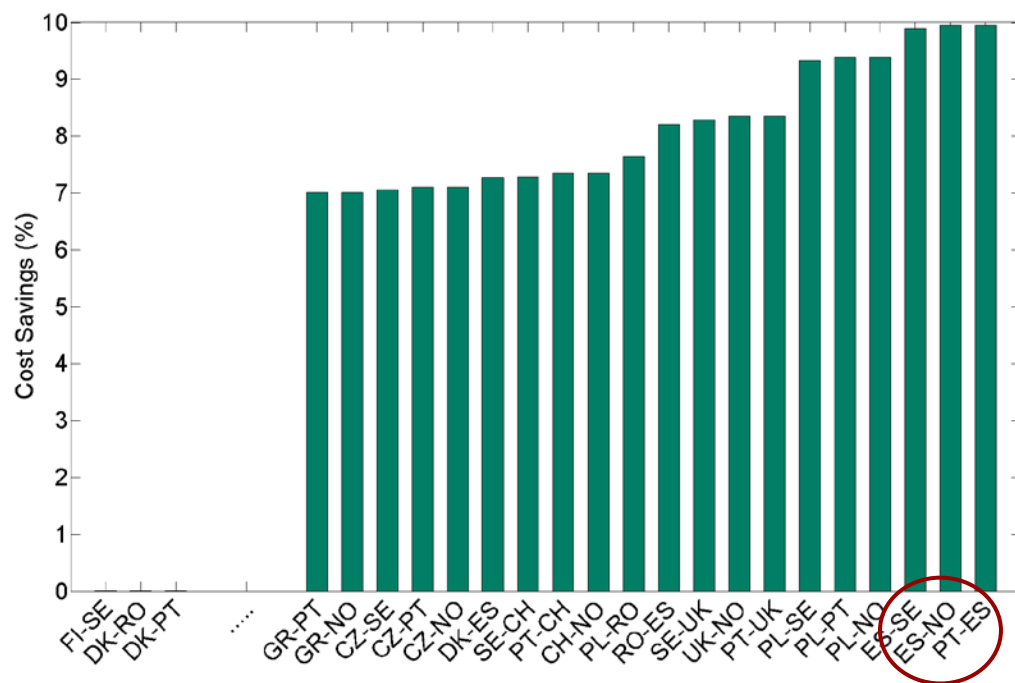


Placement of 1<sup>st</sup> Line

$$\max\{ c_{HC} \cdot \Delta P \}$$



# Verification through OPF runs



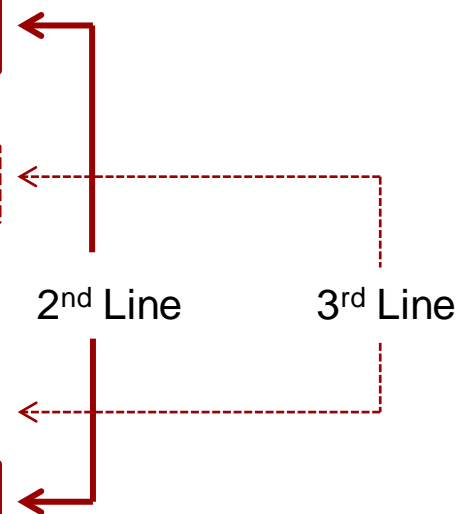
- Run 496 instances of OPF for each possible placement between the 32 nodes
- Lines PT-ES and NO-ES resulted in the highest cost savings: 9.95%

# OPF Results – Marginal Generators

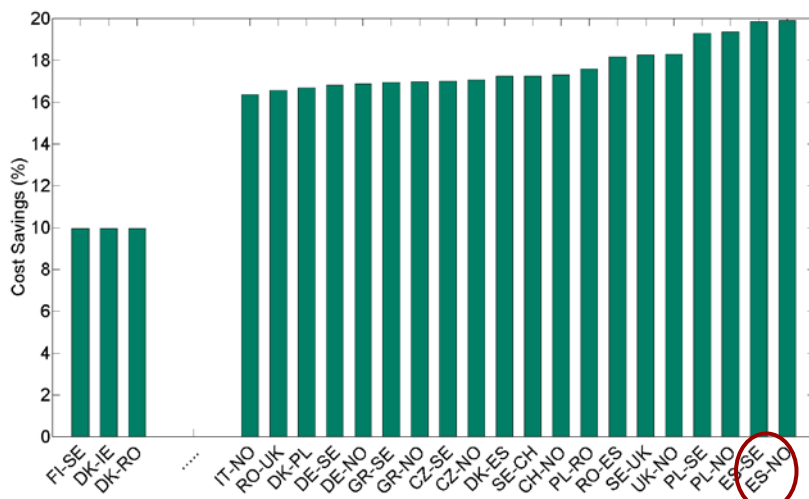
## Placement of 2<sup>nd</sup> and 3<sup>rd</sup> HVDC connection

	P <sub>g</sub> (MW)	P <sub>g,max</sub> (MW)	Costs (€/MWh)
...	...	...	...
Ireland	7'230	10'730	0.01
Portugal	26'210	30'640	0.01
Romania	9'040	10'840	0.01
Norway	33'720	44'060	0.01
France	52'940	54'280	0.54
Sweden	1'530	11'450	0.55
...	...	...	...
Hungary	830	9'300	129.05
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- Placement of 10'000 MW HVDC

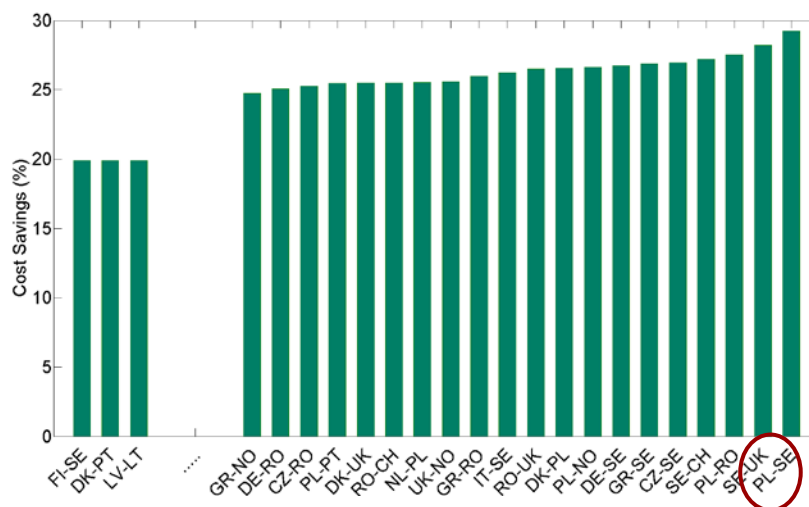


# Verification through OPF



- OPF Results fully confirm the findings through the analytical approach

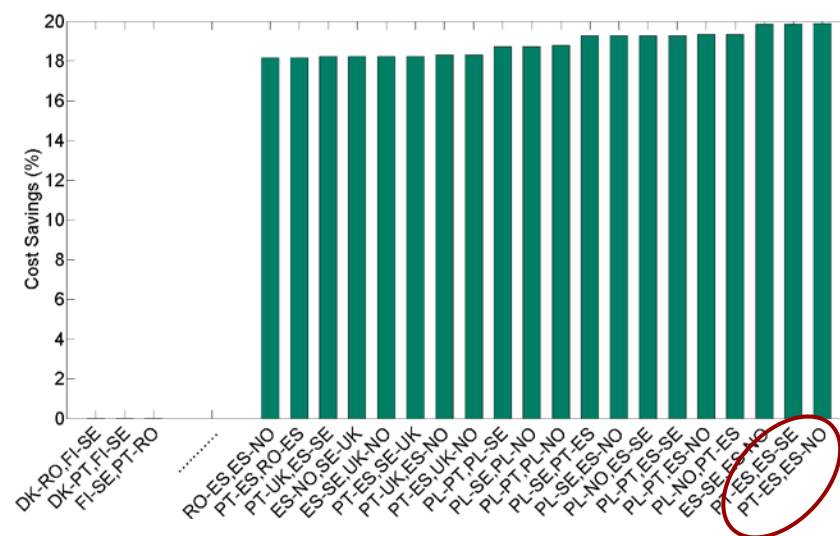
- 2<sup>nd</sup> HVDC: Spain-Norway  
Total Savings: 19.90%



- 3<sup>rd</sup> HVDC: Poland-Sweden  
Total Savings: 29.23%

# Simultaneous Placement

- Proposed algorithm followed a sequential approach
- Simultaneous placement: run 122'760 instances of OPF for placing simultaneously two HVDC connections
- Results from simultaneous placement are exactly the same as for sequential placement



Maximum Cost Savings for:

- simultaneous HVDC placement:  
PT-ES and NO-ES
- Sequential HVDC placement:  
1<sup>st</sup>: PT-ES, 2<sup>nd</sup>: NO-ES

# Conclusions

- Placing an HVDC line between a high-cost and a low-cost marginal generator leads to the maximization of the social welfare
- *Upper bound*: any line with installation costs higher than the line connecting two marginal generators should be discarded
- Algorithm for sequential placement based on the analytical approach

## Additional findings

- For  $N$  congestions there exist exactly  $N+1$  marginal generators (*assumption: DC-OPF, linear generator costs*)
- Line LMs only dependent on costs of marginal generators
  - Line LMs are an index of how congestion relief affects social welfare
- Lower bound on the expected cost savings

# Thank you!



[www.irene-40.eu](http://www.irene-40.eu)

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# For N congestions there are N+1 marginal generators

$$\min \sum_{i=1}^{N_{PG}} c_i P_{G,i}$$

DC-OPF Formulation based on PTDFs

$$|P_G - P_L| \leq F_L$$

$$0 \leq P_G \leq P_{G,max}$$

$$\mathcal{L}(P_G, \nu, \lambda, \mu) = \sum_{i=1}^{N_{PG}} c_i P_{G,i} + \nu \cdot \left( \sum_{i=1}^{N_{PG}} P_{G,i} - \sum_{i=1}^{N_{PL}} P_{L,i} \right) + \sum_{k=1}^{N_L} \lambda_k^+ \cdot (P_G - P_L) - F_{L,k} + \sum_{i=1}^{N_{PG}} (\mu_i^+ \cdot (P_{G,i} - P_{G,i,max})) + \sum_{i=1}^{N_{PG}} (\mu_i^- \cdot (-P_{G,i}))$$

Lagrangian Function of the Problem

due to KKT conditions

$$\frac{\partial \mathcal{L}}{\partial P_G} = 0$$

for one congested line

Two equations and two unknowns

$$c_m + \nu + \lambda_k^+ \cdot PTDF_{k,m} = 0$$

$$c_n + \nu + \lambda_k^+ \cdot PTDF_{k,n} = 0$$

LM of equality constraint      LM of congested line

## What if we had three marginal generators for one congested line?

Three equations  
for two unknowns

$$\left\{ \begin{array}{l} c_m + \nu + \lambda_k^+ \cdot PTDF_{k,m} = 0 \\ c_n + \nu + \lambda_k^+ \cdot PTDF_{k,n} = 0 \\ c_p + \nu + \lambda_k^+ \cdot PTDF_{k,p} = 0 \end{array} \right.$$

- In order to exist such a solution, the third marginal generator should satisfy a certain relationship between generator costs and PTDF factors ( $\sim$ network topology):

$$c_p = c_m + \frac{c_m - c_n}{PTDF_{k,n} - PTDF_{k,m}} \cdot (PTDF_{k,m} - PTDF_{k,p})$$

Such a “coincidence” is very difficult to occur in real power systems



Therefore, in most power systems for N congestions there are N+1 marginal generators

# Elimination of Marginal Generators = Congestion Relief

- Congestion relief = set  $\lambda_k$  to zero
- Lagrangian Multipliers  $\lambda_k$  of congested lines are dependent only on the costs of the marginal generators

One congested line:

$$\lambda_k = \xi_k \cdot \frac{c_m - c_n}{PTDF_{k,n} - PTDF_{k,m}}$$

Annotations in the diagram:

- Blue box: Costs of the two marginal generators (points to  $c_m$  and  $c_n$ )
- White box: Direction of the flow (points to  $\xi_k$ )
- White box: Dependent on network topology (points to  $PTDF_{k,n}$  and  $PTDF_{k,m}$ )

For more congested lines:

$$\begin{bmatrix} 1 & \xi_{k1} \cdot PTDF_{k1,g1} & \xi_{k2} \cdot PTDF_{k2,g1} \\ 1 & \xi_{k1} \cdot PTDF_{k1,g2} & \xi_{k2} \cdot PTDF_{k2,g2} \\ 1 & \xi_{k1} \cdot PTDF_{k1,g3} & \xi_{k2} \cdot PTDF_{k2,g3} \end{bmatrix} \cdot \begin{bmatrix} \nu \\ \lambda_{k1} \\ \lambda_{k2} \end{bmatrix} = - \begin{bmatrix} c_{g1} \\ c_{g2} \\ c_{g3} \end{bmatrix}$$

Annotation in the diagram:

- Blue box: marginal generators (points to the cost vector  $\begin{bmatrix} c_{g1} \\ c_{g2} \\ c_{g3} \end{bmatrix}$ )