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Remedial Actions to Enhance Stability of Low-Inertia Systems

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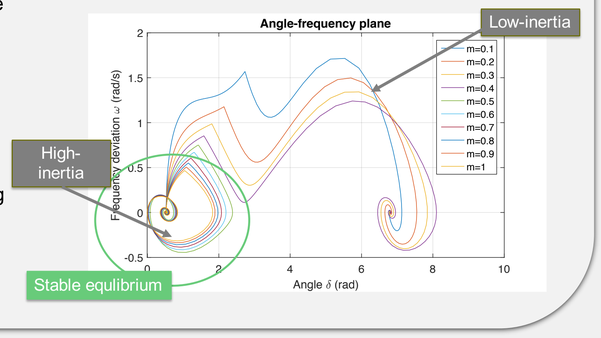
Goal #1: Enhance transient stability of low-inertia systems through inertia and damping control

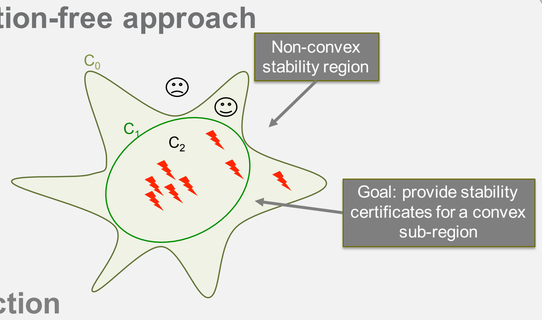
- Low inertia systems are more prone to instability.
- Increasing inertia and damping control during the fault-on dynamics increases the region of attraction
- Synthetic inertia and damping can be provided by external sources

High-inertia systems are more probable to converge back to the stable equilibrium for the same disturbance



Goal #2: Use a simulation-free approach

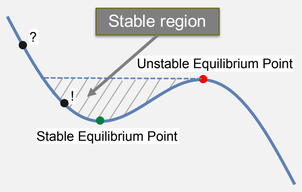
- Stability certificates: tractable sufficient conditions
- Strategy: certify security of most of scenarios with conservative conditions, use simulations for few really dangerous scenarios



Example: Energy Function

generator frequencies

$$E = \underbrace{\frac{1}{2}\dot{\delta}^T M \dot{\delta}}_{\text{Kinetic energy}} - \underbrace{\sum \kappa_{kj} \cos \delta_{kj} - P^T \delta}_{\text{Potential energy}}$$

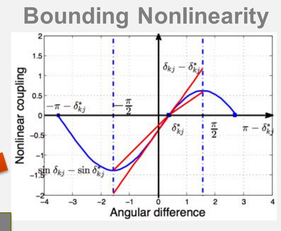


Quadratic Lyapunov Functions for Transient Stability

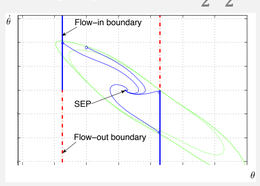
$$\dot{x} = Ax - BF(Cx)$$

- Structure-preserving model
- $x = \delta_i - \delta_i^*$
- A, B, C independent of operating point
- Non-linear model: $F(Cx) = \sin \delta_{kj} - \sin \delta_{kj}^*$

Bounding Nonlinearity



Polytope $P \in [-\frac{\pi}{2}, \frac{\pi}{2}]$



From [2], if:

$$\bar{A}^T P + P\bar{A} + \frac{(1-g)^2}{4} C^T C + PBB^T P \preceq 0$$

There exists a quadratic Lyapunov function:

$$V = x^T P x$$

that is decreasing whenever $x(t)$ is inside polytope P

Contributions

Exact reformulation to relax the LMI condition

$$\dot{x} = Ax - B\Lambda^{-1}\Lambda F(Cx)$$

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} + \frac{(1-g)^2}{4} C^T Q C & PB \\ B^T P & -Q \end{bmatrix} \leq 0$$

- $Q = \Lambda^T \Lambda$
- Freely determine diagonal $Q > 0$

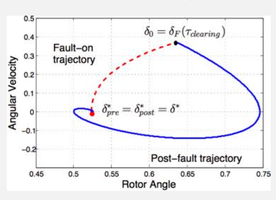
Optimal tuning of inertia and damping

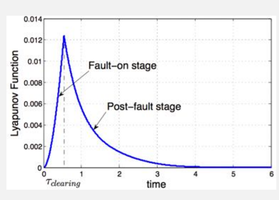
- Assumption: There is a fault for which we cannot find a P
- Goal: Find (m,d) which decrease the growth rate of a Lyapunov function $x^T \tilde{P} x$, so that the fault remains inside the ellipsoid

$$\begin{bmatrix} \bar{A}(m,d)^T \tilde{P} + \tilde{P}\bar{A}(m,d) + \frac{(1-g)^2}{4} C^T Q C & \tilde{P}\bar{B}(m,d) \\ \bar{B}(m,d)^T \tilde{P} & -Q \end{bmatrix} \leq 0$$

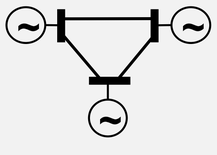
- Problem is bilinear: Find (m,d) and $\tilde{P}$
- Alternate between two optimization problems: Fix (m,d) and find $\tilde{P}$ ⟷ Fix $\tilde{P}$ and *min(m,d)*

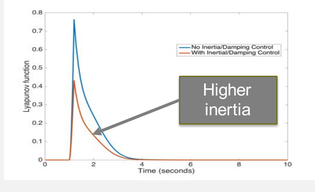
Numerical Example





Example: 3-machine system





- Higher inertia and damping limit the growth rate of the Lyapunov function
- Fault-cleared state remains inside the stability region

Conclusions

- Presented stability and resiliency certificates that are:
  - Robust: independent of the operating point (within bounds)
  - Convex: quadratic functions
  - Less conservative potentially: it's a family of Lyapunov functions
- Introduced exact reformulation to relax the problem and obtain better results
- Incorporated remedial actions:
  - Use of external power sources to mimic inertia and damping
  - Simulation-free stability guarantees for a larger set of faults
  - Low energy and power requirements: external sources act only during fault-on dynamics

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[1] T.Vu and K.Turitsyn,"Lyapunov functions family approach to transient stability assessment," *Power Systems, IEEE Transactions on*, vol. PP, no. 99, pp. 1–9, 2015.
[2] T. L. Vu and K. Turitsyn, "A Framework for Robust Assessment of Power Grid Stability and Resiliency," *Automatic Control, IEEE Trans.*, 2016