Technical University of Denmark

multiDC

Market integration of HVDC

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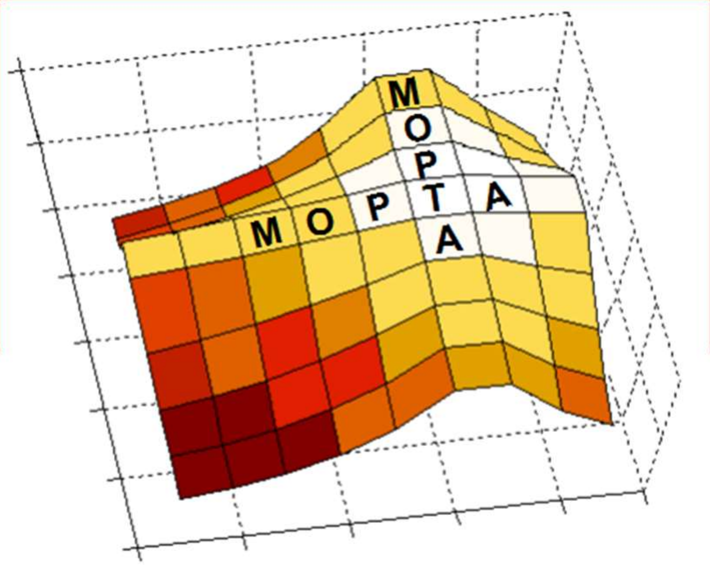

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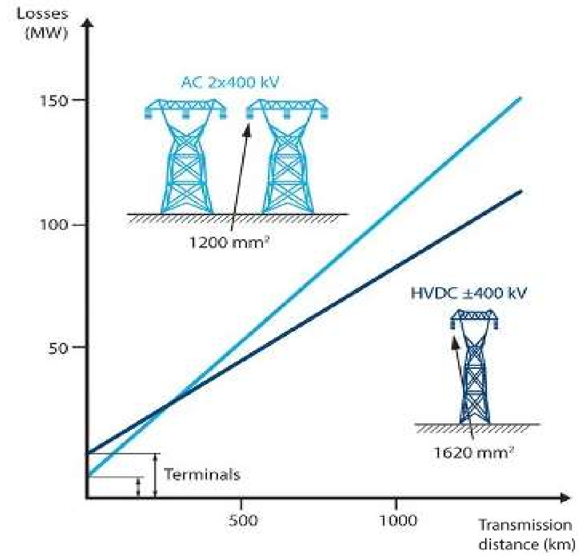
August 17, 2018

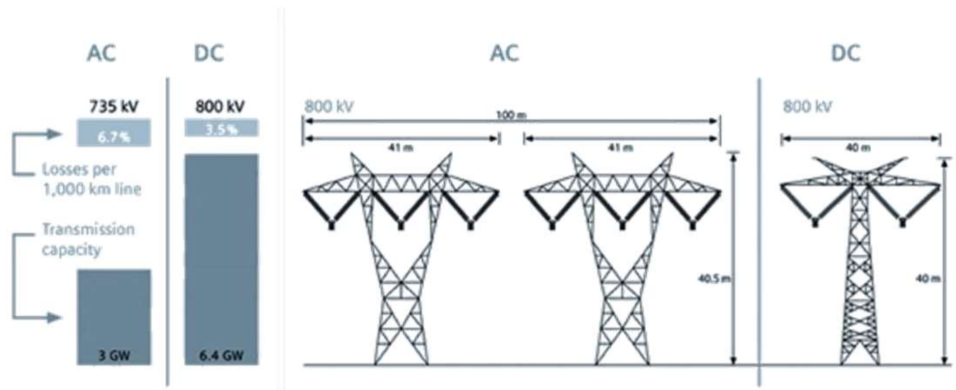
High-Voltage Direct-Current lines

HVDC transmission systems use **direct current** for transmission of electrical power, in contrast with the more common **alternating current** (AC) systems

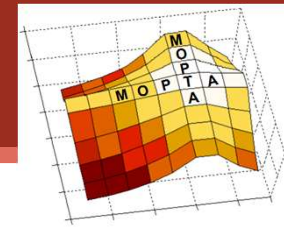


Benefit of HVDC:

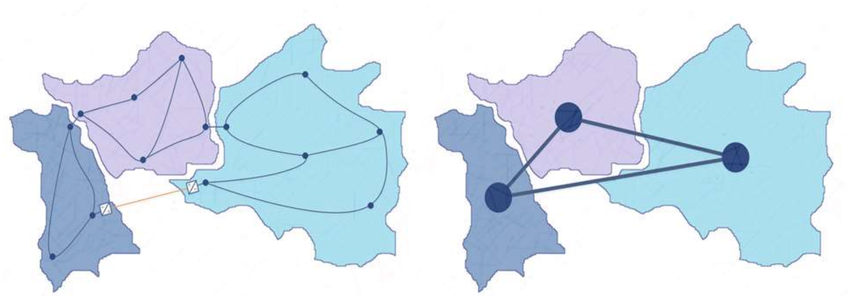
- **Economic** power transmission over long distances
- Connection of **asynchronous** networks
- Full **controllability** of the power flows
- Environmental friendly technology (lower visual impact, grid access for renewable,...)

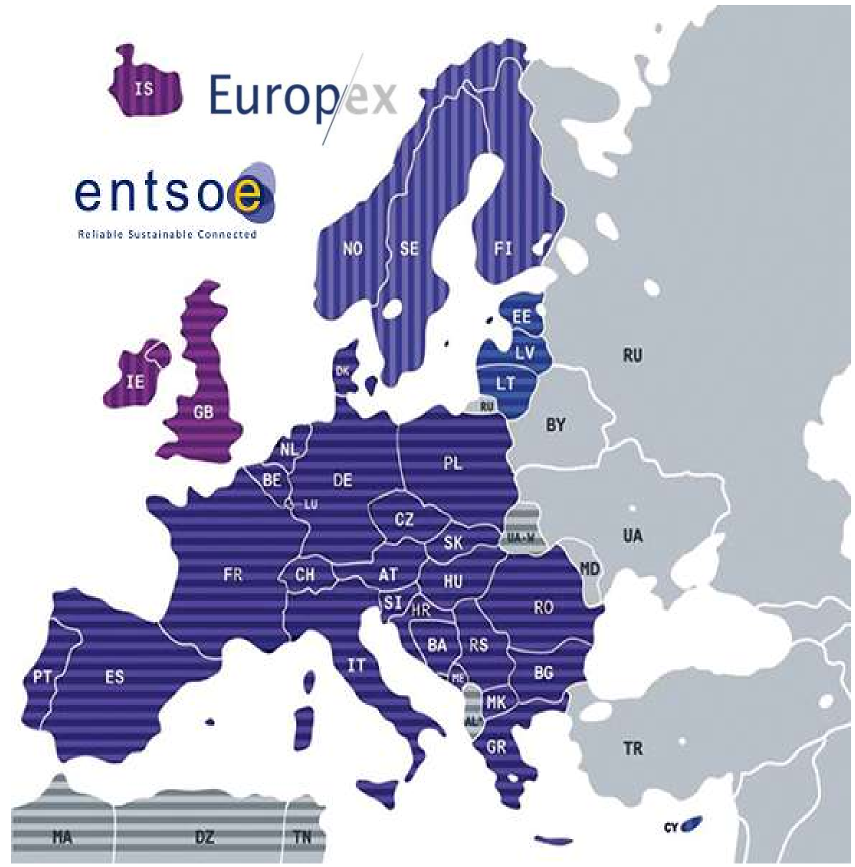


Internal European Electricity Market

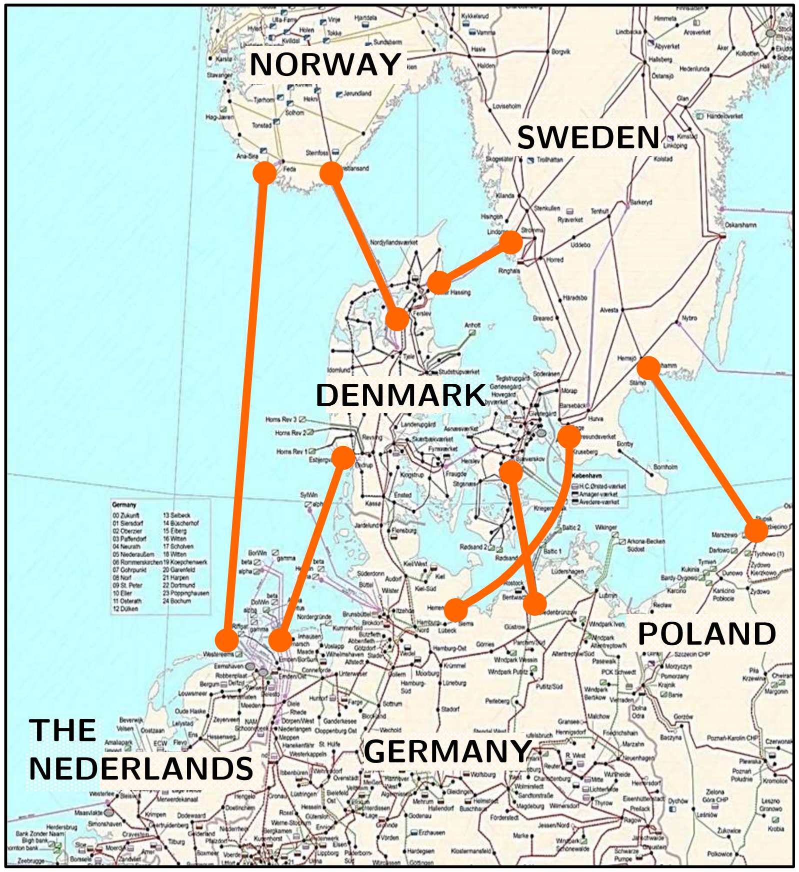


- **Flow-based market coupling:** each country is considered as a copperplate and the interconnectors are modelled as a single equivalent flowgate.
- Price differences minimized, convergence if sufficient capacity
- Efficient use of interconnector capacity





Motivation



- **HVDC interconnectors** are usually longer than **AC** interconnectors.
- For this reason, the losses occurring on HVDC lines are not negligible.
- If **price difference** between areas is **small**, TSOs cannot recover the cost of HVDC losses.
  - Cost of losses higher than potential revenue
- Introduction of an **HVDC loss factor** in the market clearing*

SVENSKA KRAFTNÄT Statnett ENERGINET FINGRID

* Fingrid, Energinet, Statnett, Svenska Kraftnät, *Analyses on the effects of implementing implicit grid losses in the Nordic CCR*, April 2018

An example: Kontiskan

NORD POOL — EL SPOT

05-07-2018	MWh/h SE3 > DK1	MWh/h DK1 > SE3	EUR/MWh SE3	EUR/MWh DK1
00 - 01	0,0	4,1	50,34	49,13
01 - 02	23,1	0,0	48,55	45,48
02 - 03	103,9	0,0	47,54	44,31
03 - 04	0,0	49,0	47,14	47,14
04 - 05	71,9	0,0	47,34	47,34
05 - 06	41,3	0,0	49,35	47,57
06 - 07	80,7	0,0	53,17	51,89
07 - 08	60,5	0,0	56,43	57,71
08 - 09	109,0	0,0	61,21	61,21
09 - 10	137,1	0,0	60,94	60,94
10 - 11	364,0	0,0	62,41	62,41
11 - 12	190,6	0,0	64,07	64,07
12 - 13	0,0	19,4	63,88	63,88
13 - 14	0,0	0,0	63,57	52,25
14 - 15	0,0	0,0	59,04	52,06
15 - 16	34,7	0,0	57,56	51,84
16 - 17	90,8	0,0	53,97	52,02
17 - 18	139,8	0,0	52,97	52,97
18 - 19	161,2	0,0	54,83	54,83
19 - 20	237,0	0,0	55,41	55,41
20 - 21	154,6	0,0	55,10	55,10
21 - 22	21,7	0,0	53,79	53,79
22 - 23	3,5	0,0	52,05	52,05
23 - 00	0,0	34,7	50,21	50,21

Source: https://www.nordpoolgroup.com/

HVDC LINK BETWEEN DENMARK AND SWEDEN

An example: Kontiskan

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› In 2018, until now, the **price difference** between **Sweden** and **Denmark** has been zero for more than 2400 hours (**54%**)...

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- In 2018, until now, the **price difference** between **Sweden** and **Denmark** has been zero for more than 2400 hours (**54%**)...
- ...and back in 2017, for more than 5300 hours (**61%**).

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- In 2018, until now, the **price difference** between **Sweden** and **Denmark** has been zero for more than 2400 hours (**54%**)...
- ...and back in 2017, for more than 5300 hours (**61%**).
- The total cost of losses during those 5300 hours was approx. **0.9 M€ for a single HVDC line**.

Problem statement

- **Losses** are handled in a different way for AC and HVDC lines.

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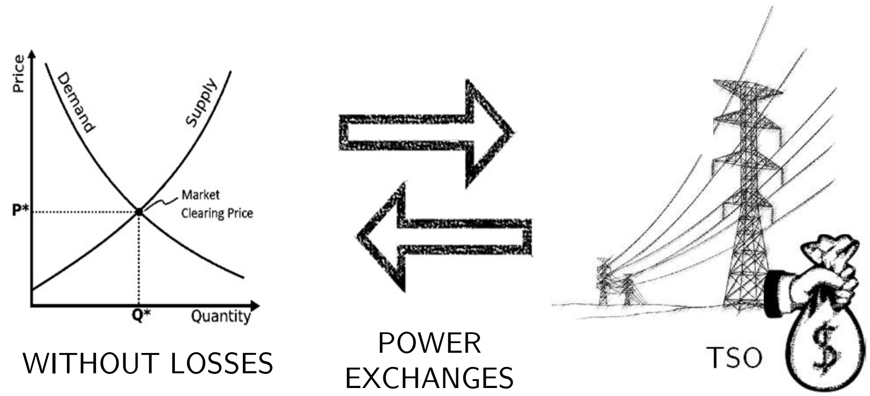
- For **HVDC lines:**

EXPLICIT METHOD



IMPLICIT METHOD

Problem statement

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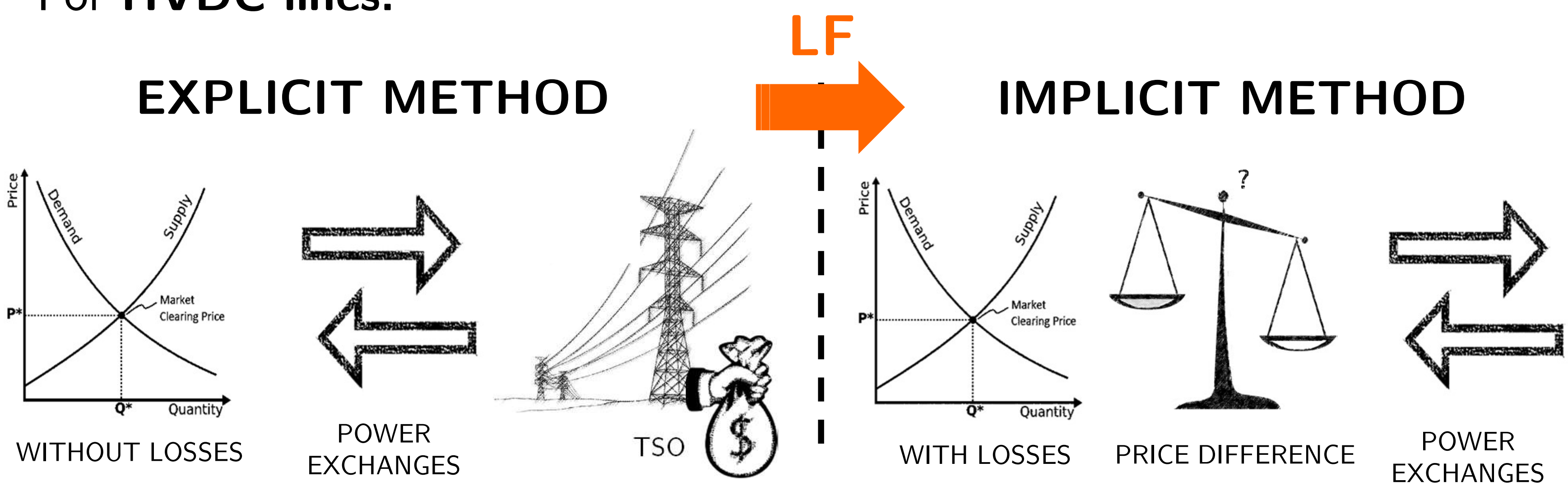

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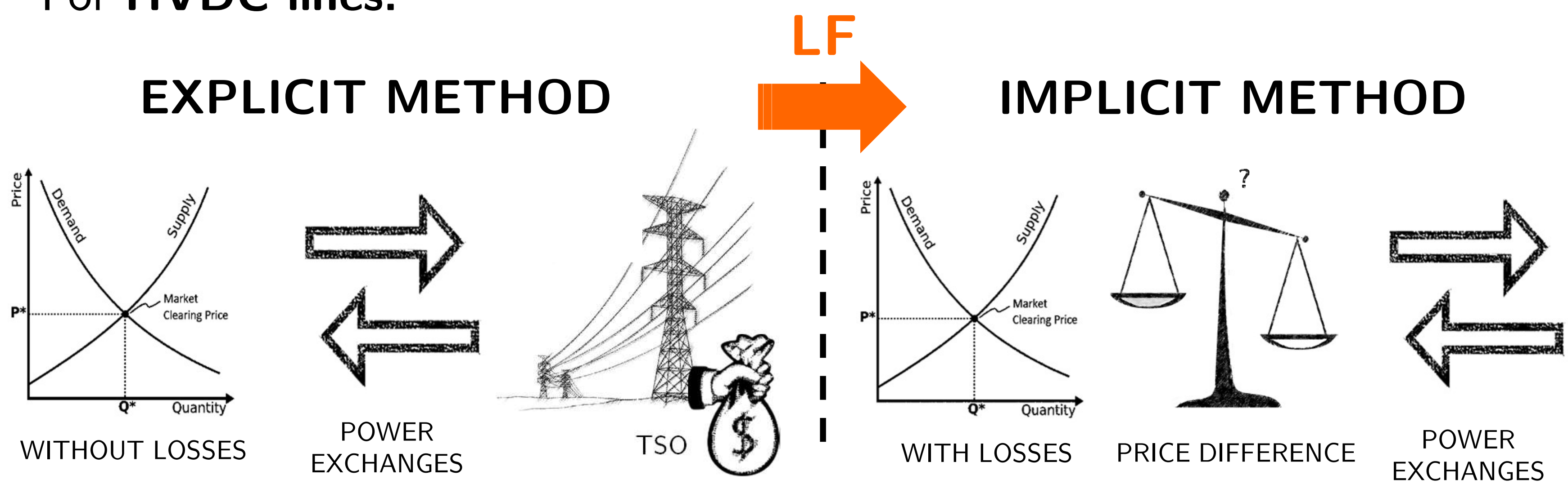
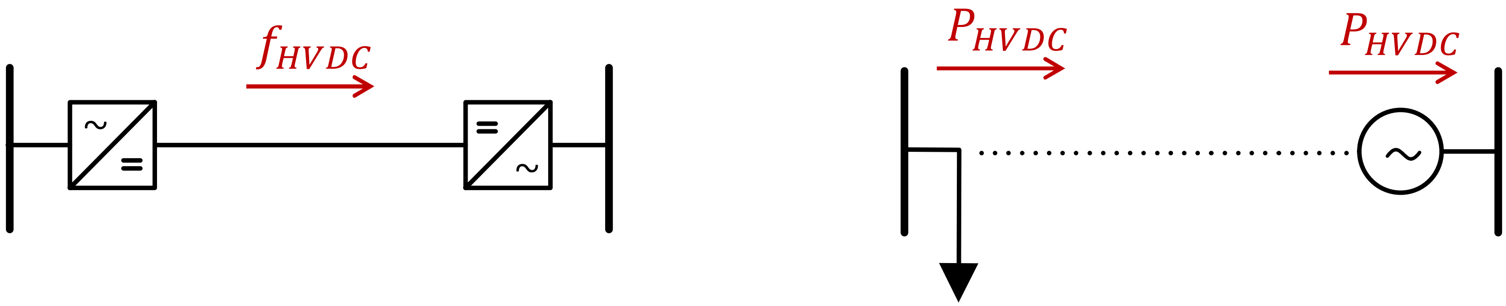


- To move from the explicit to the implicit method, a **loss factor** has to be included in the market clearing algorithm.
- **Is it a good idea to introduce a loss factor <u>only</u> for HVDC lines in meshed grids?**

HVDC line model

- Simple DC model



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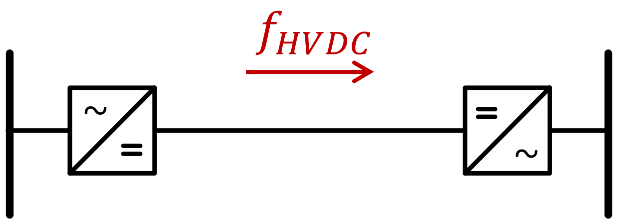

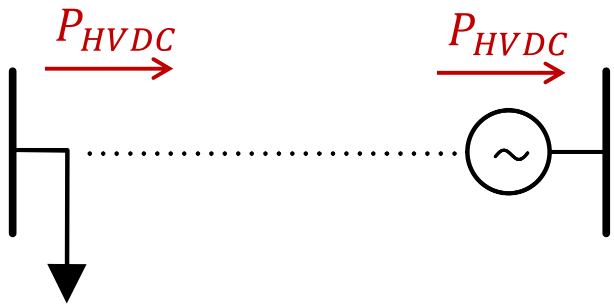

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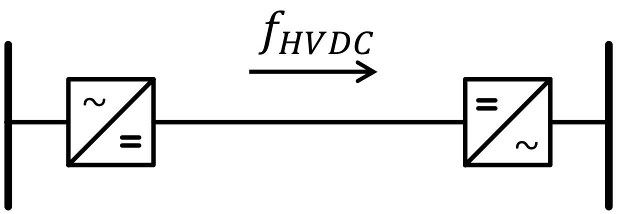

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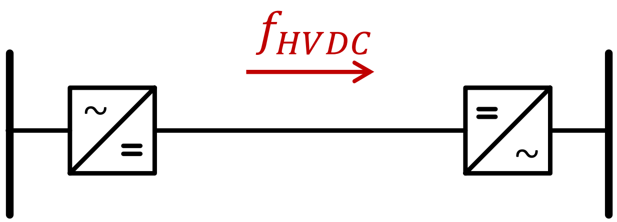
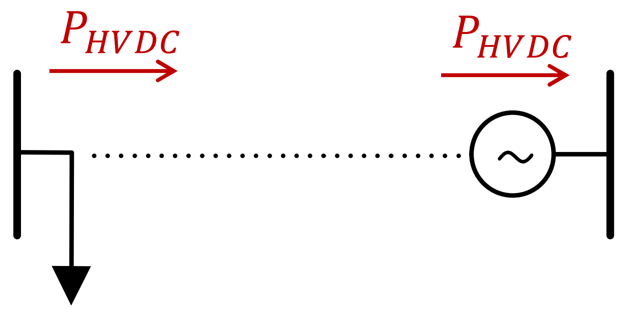
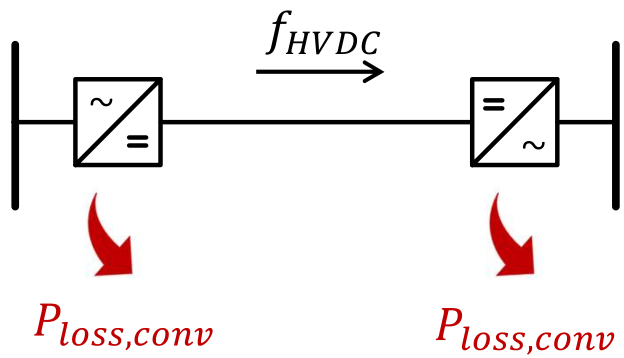




- Losses
  - **Converter station**: switching and conduction losses of power electronics components, plus losses on all the other devices.

$$P_{loss,conv} = af_{HVDC}^2 + bf_{HVDC} + c$$



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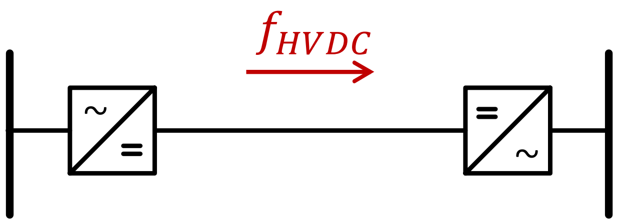

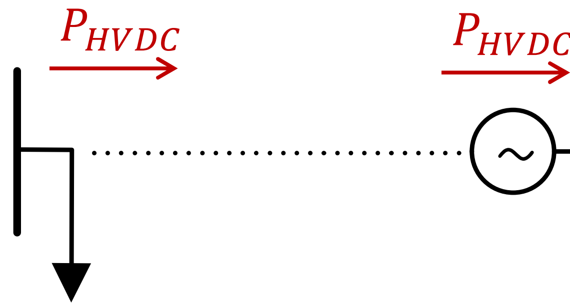

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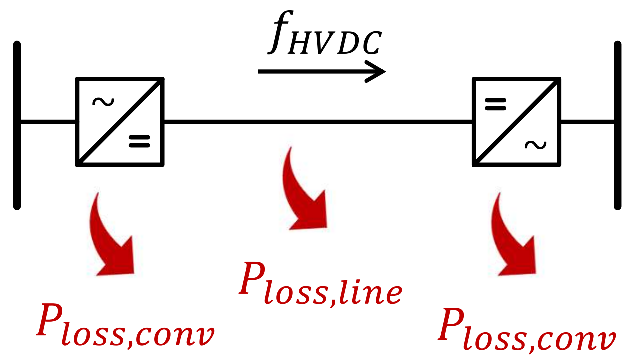
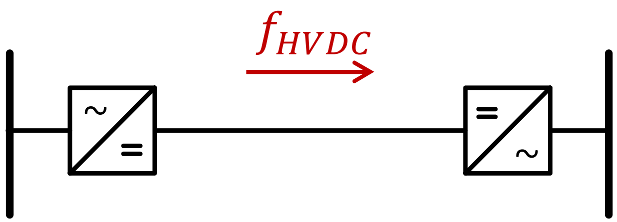


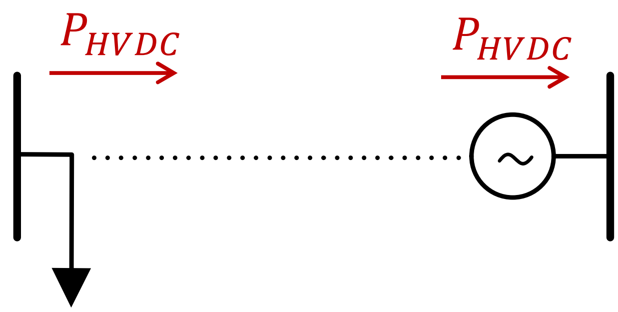
  - **Line**: conduction losses due to the resistance of the cable

$$P_{loss,line} = Rf_{HVDC}^2$$

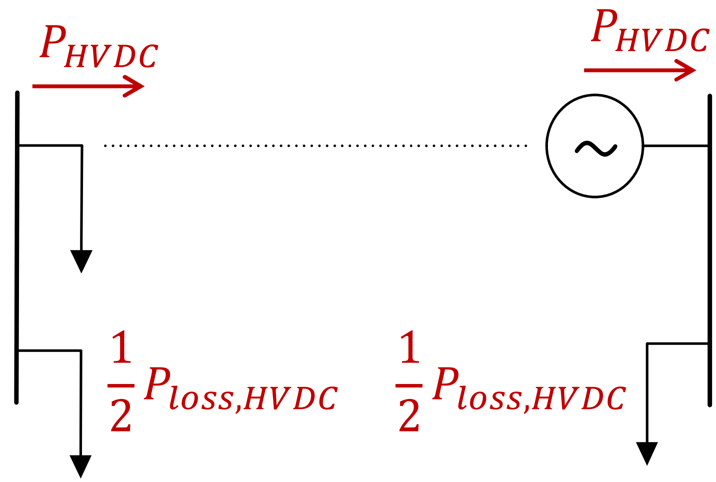
HVDC line model

- Simple DC model





- Losses
  - Losses are considered as an **extra load equally shared** by the sending and the receiving node



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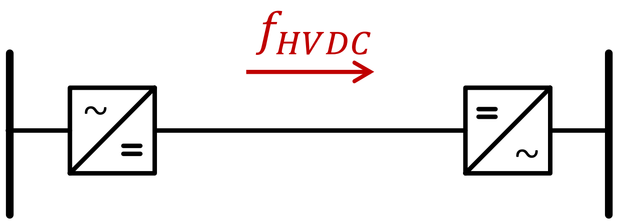
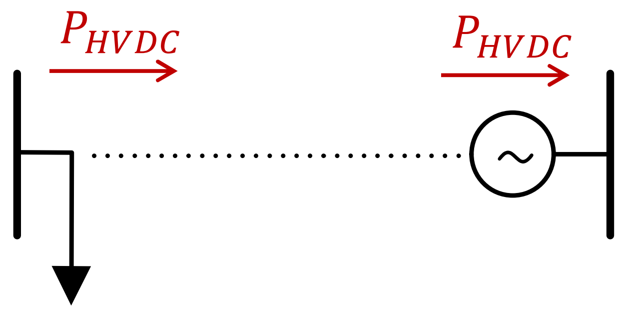
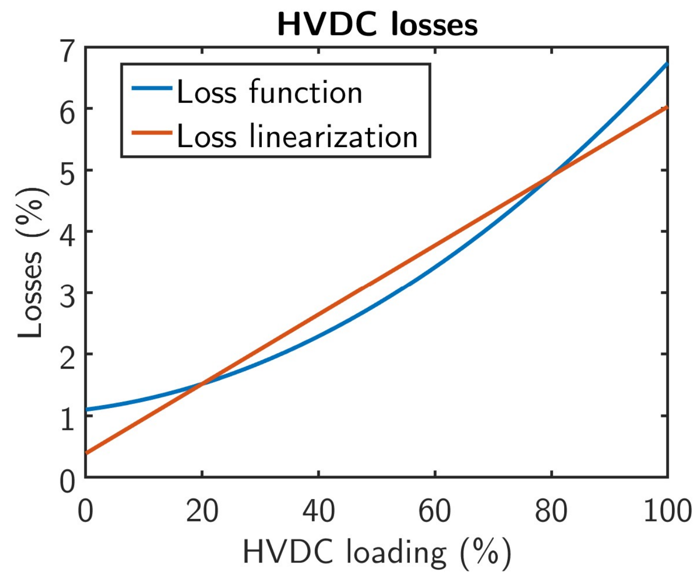




- Losses
  - Losses are considered as an **extra load equally shared** by the sending and the receiving node
  - **Linearization** of losses for their introduction in the market clearing algorithm



Introduction of the HVDC loss factor

The **DC-OPF** algorithm for clearing the market has the following form:

min	*Total system cost*	
s. t.	*Generation constraints*	
	AC line limits	
	HVDC line limits	
	Nodal balance equations	$\boldsymbol{d} + \mathbf{I}\boldsymbol{f}_{HVDC} + \mathbf{B}_{bus}\boldsymbol{\theta} - \boldsymbol{g} = 0$

Introduction of the HVDC loss factor

We introduce the **loss factor** to account for losses, and add the **losses** in the nodal balance equation:

min *Total system cost*

s.t. *Generation constraints*

AC line limits

HVDC line limits

HVDC losses $p_{lossLINE,l} \geq \alpha_l f_{HVDC,l} + \beta_l \quad \forall l$

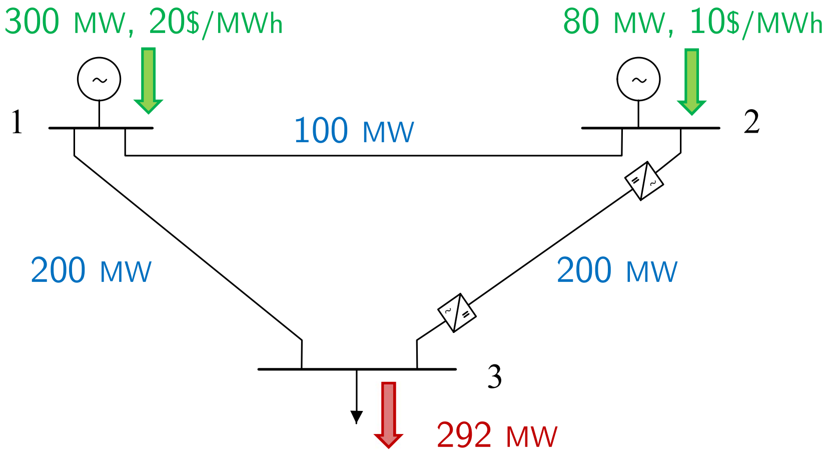
$p_{lossLINE,l} \geq \alpha_l \left(-f_{HVDC,l}\right) + \beta_l \quad \forall l$

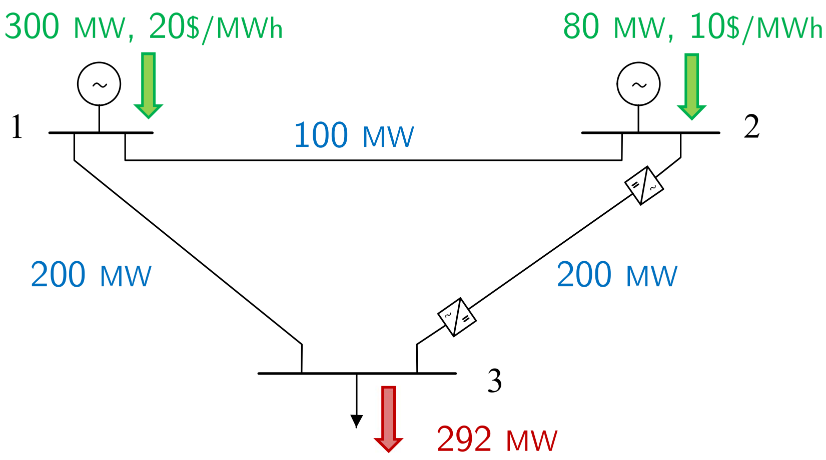
Nodal distribution $\boldsymbol{p}_{lossNODE} = \mathbf{N}\boldsymbol{p}_{lossLINE}$

Nodal balance equations $\boldsymbol{d} + \mathbf{I}\boldsymbol{f}_{HVDC} + \mathbf{B}_{bus}\boldsymbol{\theta} - \boldsymbol{g} + \boldsymbol{p}_{lossNODE} = 0$

Now the generation is equal to the sum of the demand plus the losses.

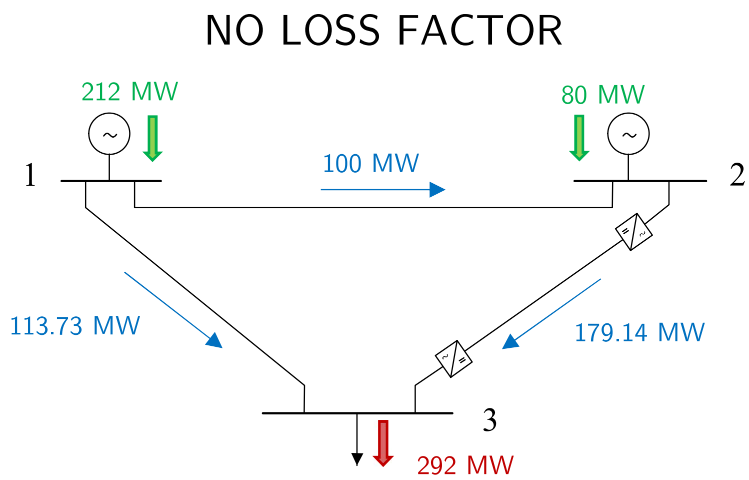
Impact on prices



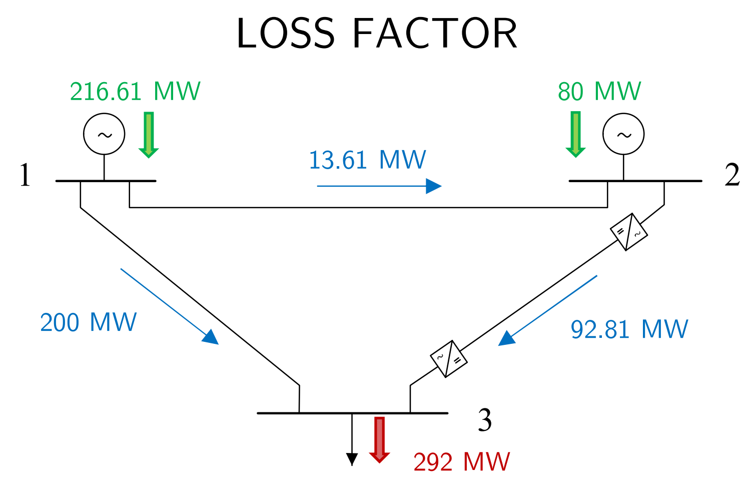


300 MW, 20$/MWh
80 MW, 10$/MWh
1
2
3
100 MW
200 MW
200 MW
292 MW

Results

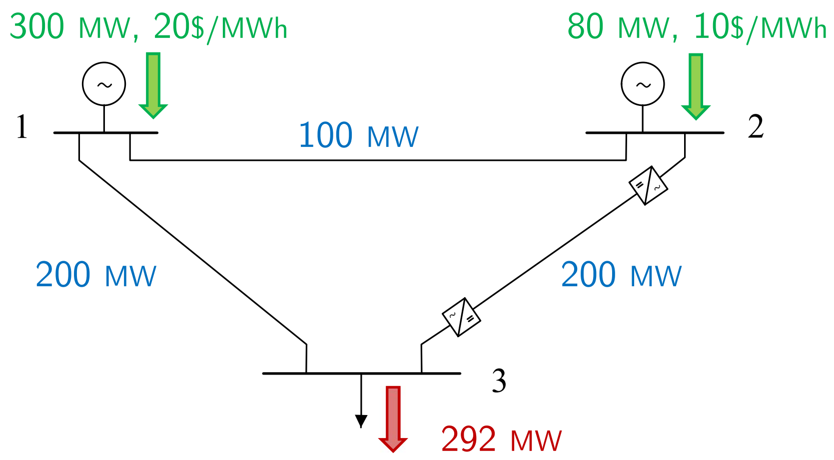


NO LOSS FACTOR
212 MW
80 MW
1
2
3
100 MW
113.73 MW
179.14 MW
292 MW



LOSS FACTOR
216.61 MW
80 MW
1
2
3
13.61 MW
200 MW
92.81 MW
292 MW

Impact on prices



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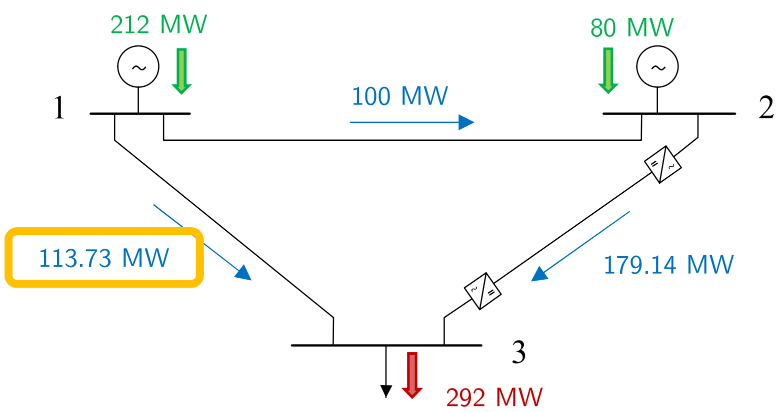
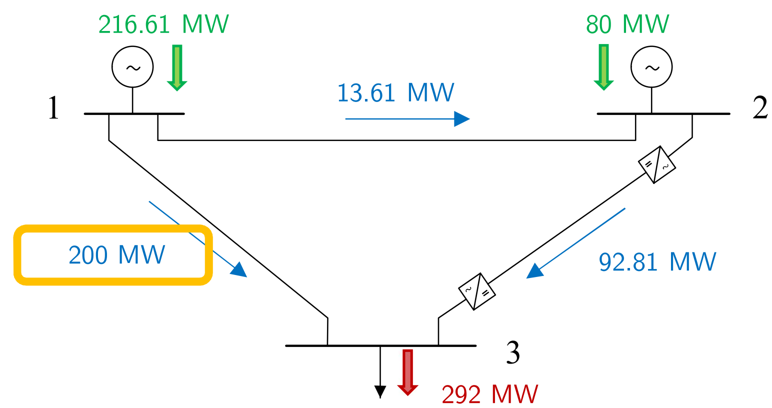


LOSS FACTOR



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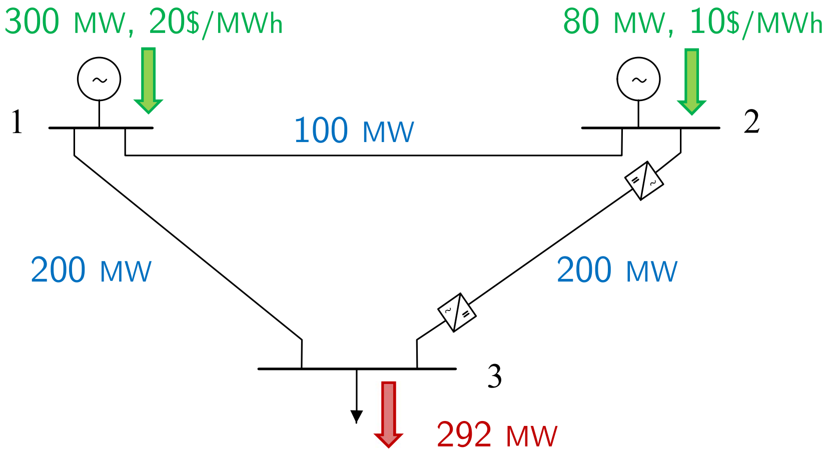
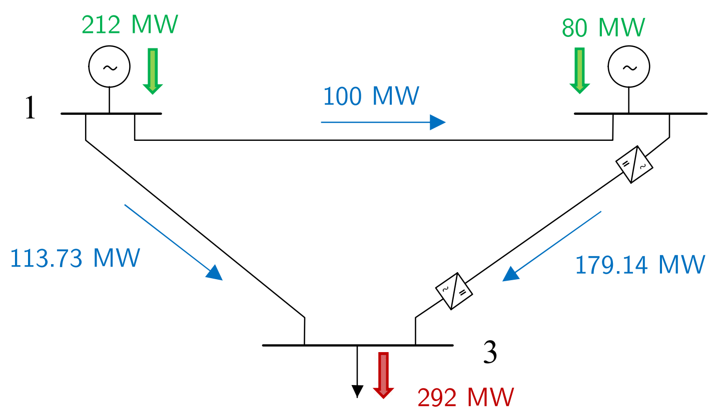


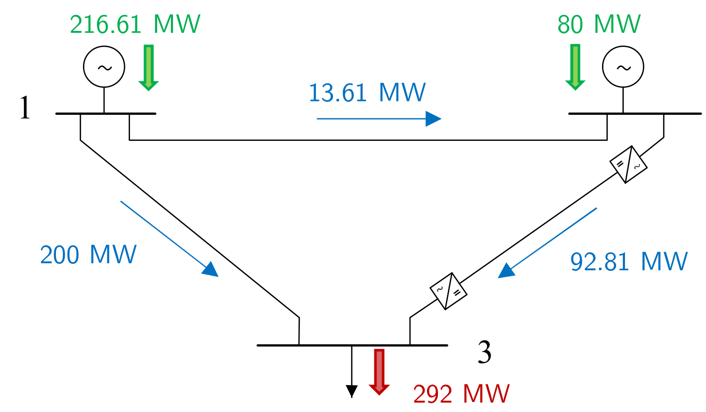
Prices	without LF ($/MWh)	with LF ($/MWh)
Zone 1	20.00	20.00
Zone 2	20.00	20.00
Zone 3	20.00	20.12

Results

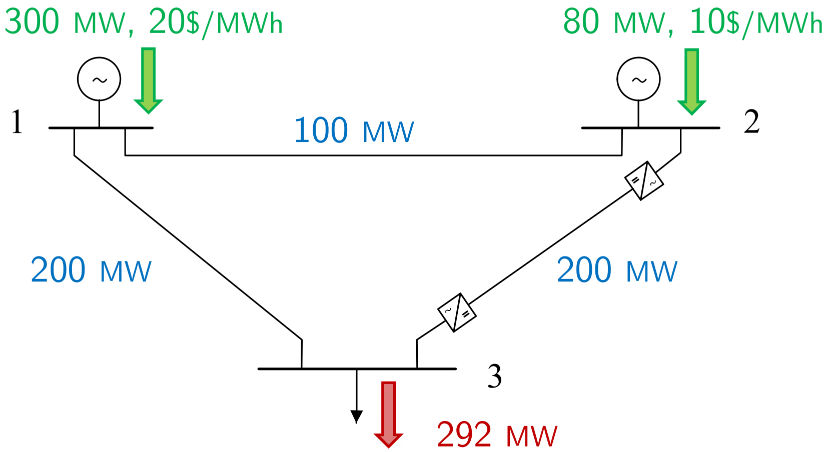
NO LOSS FACTOR



LOSS FACTOR



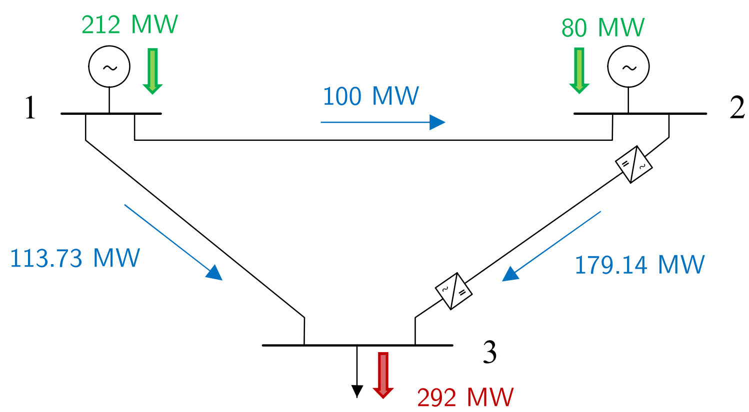
Impact on prices



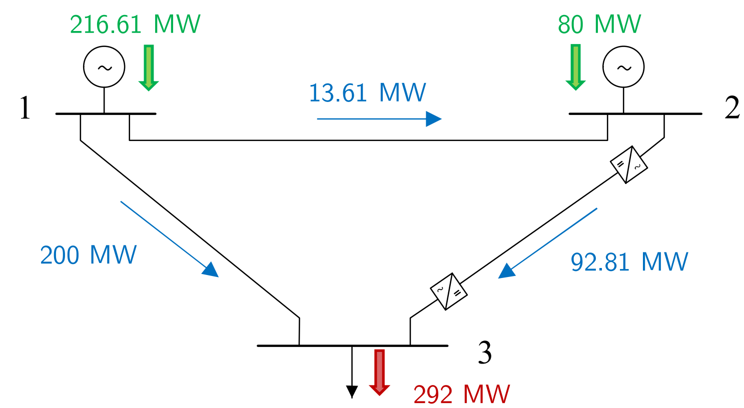
Prices	without LF ($/MWh)	with LF ($/MWh)
Zone 1	20.00	20.00
Zone 2	20.00	20.00
Zone 3	20.00	20.12

Results

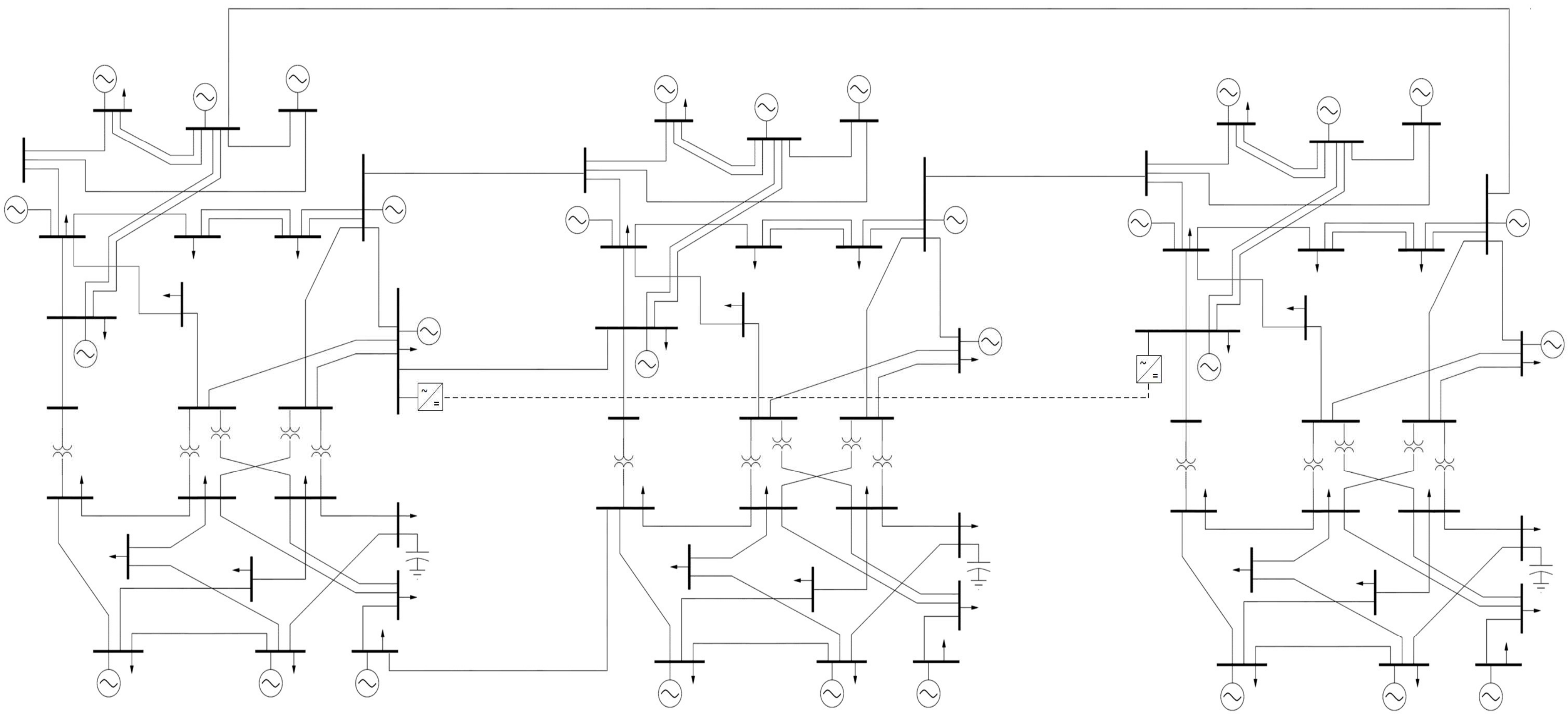
NO LOSS FACTOR



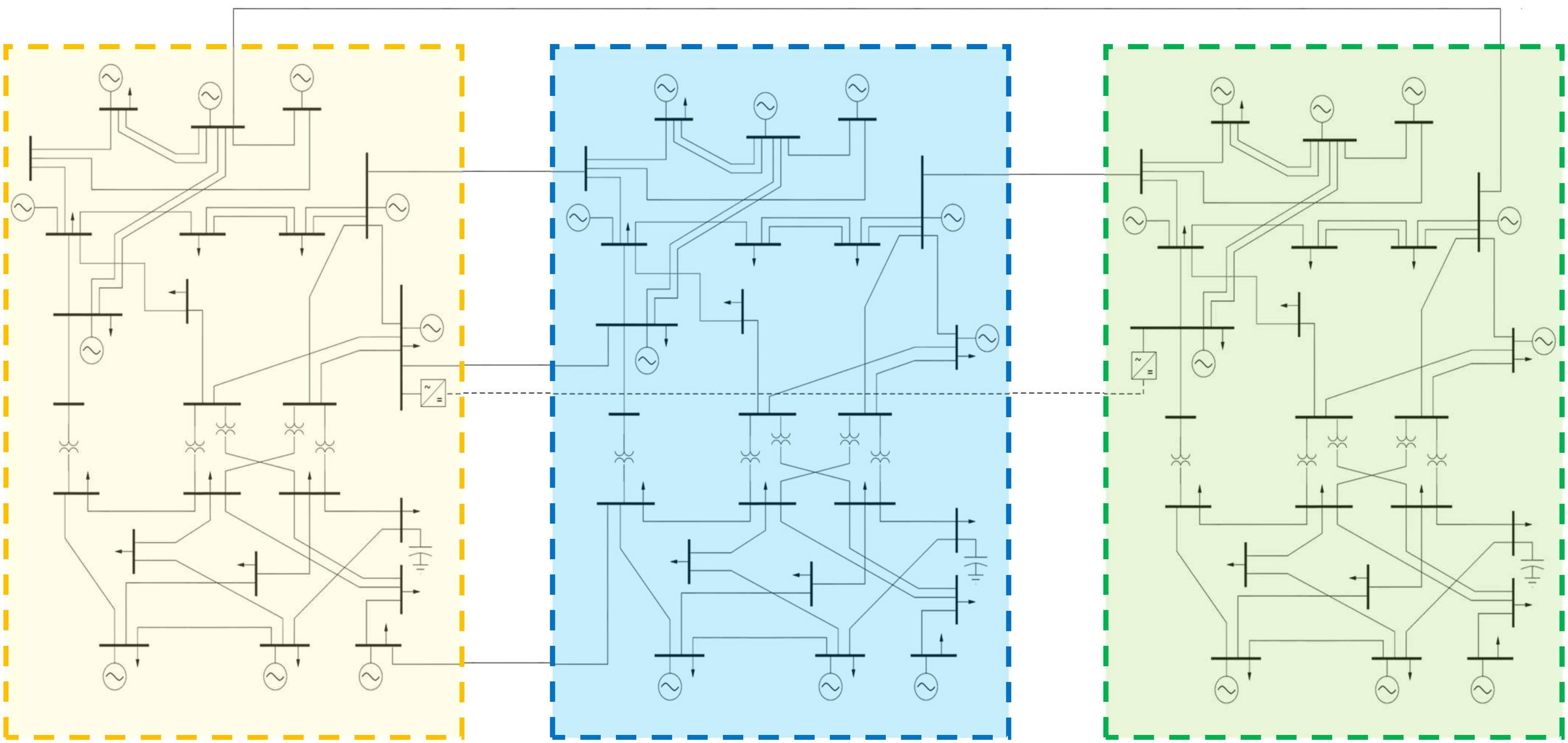
LOSS FACTOR



Test case: 3-area IEEE RTS system



Test case: 3-area IEEE RTS system

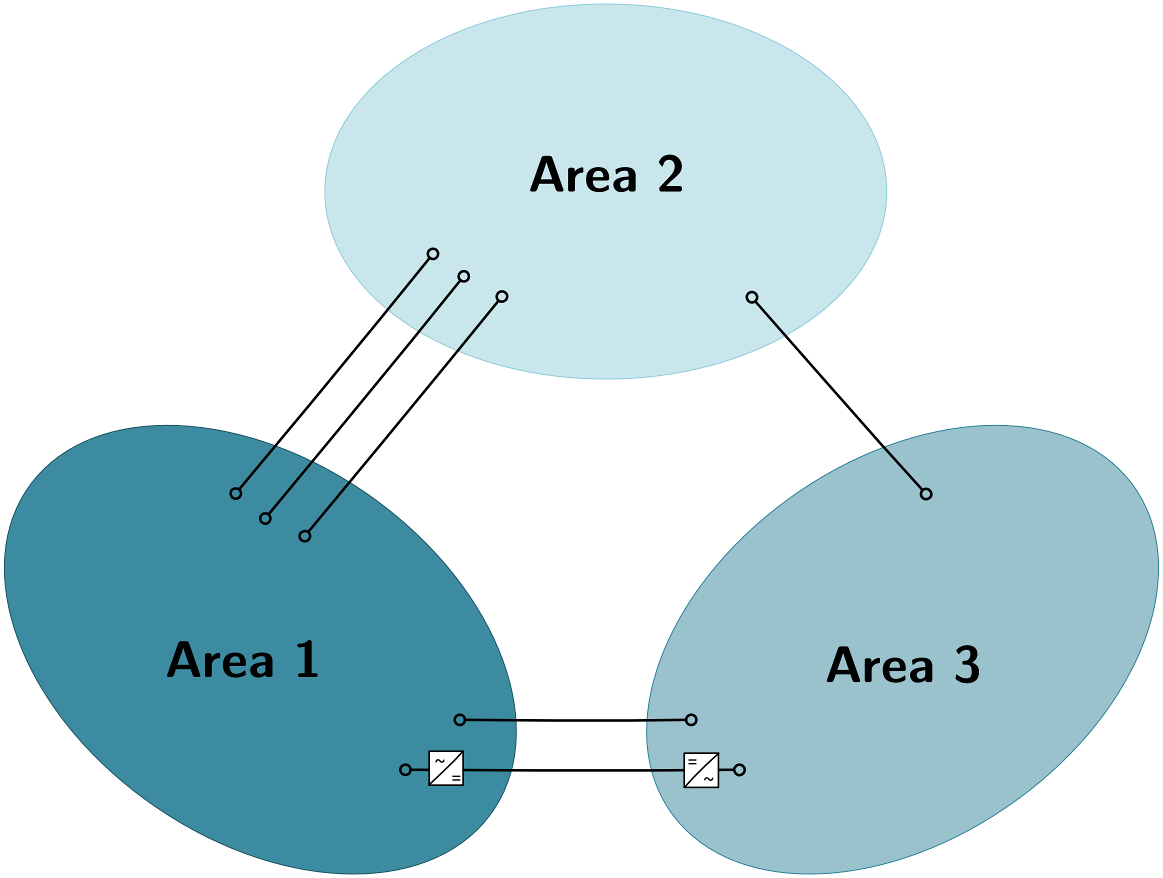


AREA 1 AREA 2 AREA 3

Flow-based market coupling

For **each area**:

- 32 producers
- 17 consumers

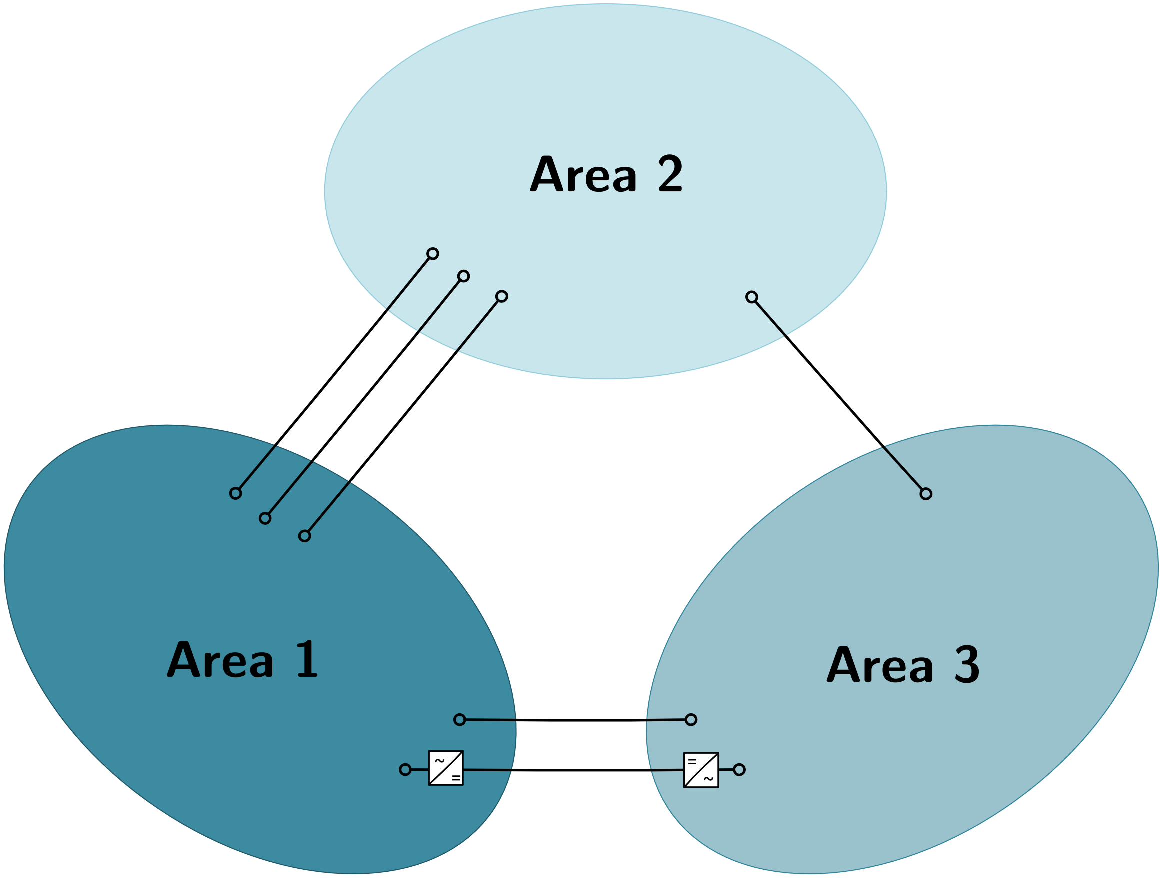


Flow-based market coupling

For **each area**:

- 32 producers
- 17 consumers

Four **different situations**:



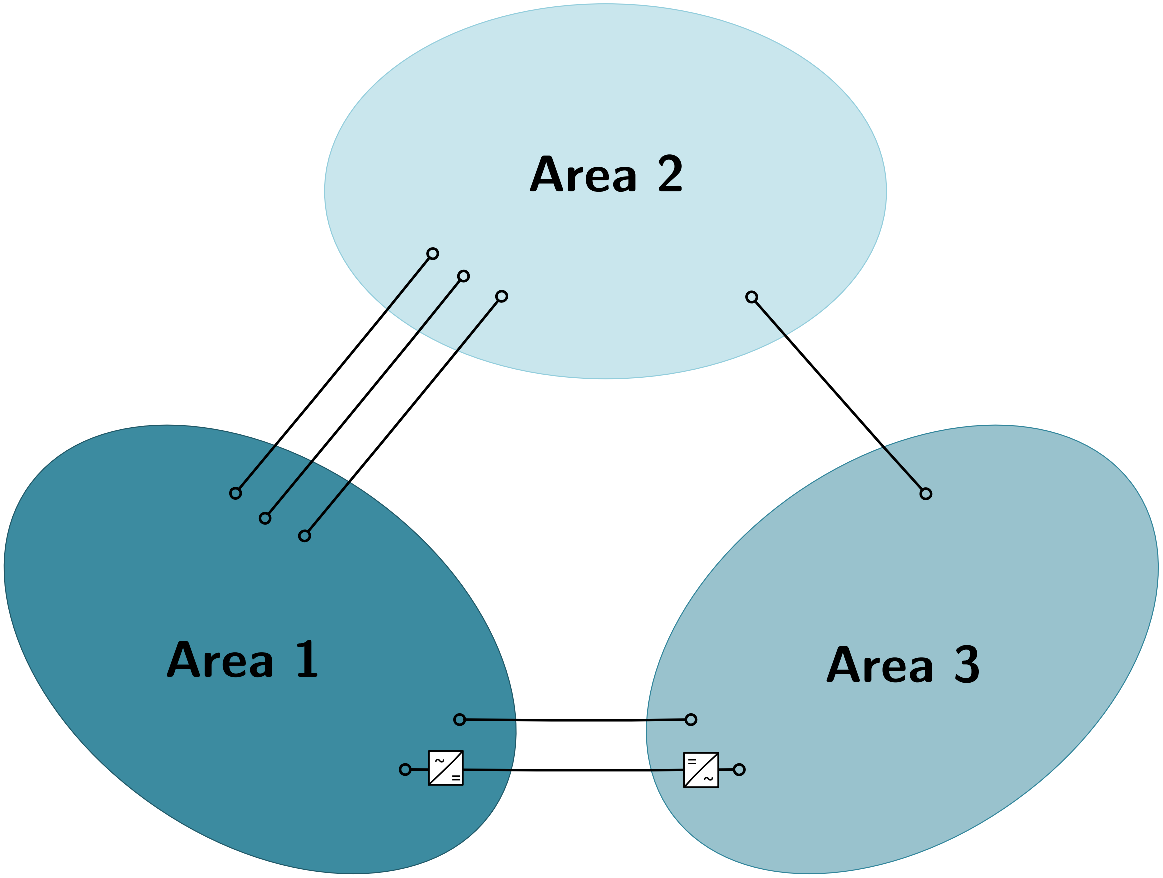
Flow-based market coupling

For **each area**:

- 32 producers
- 17 consumers

Four **different situations**:

- Normal operation



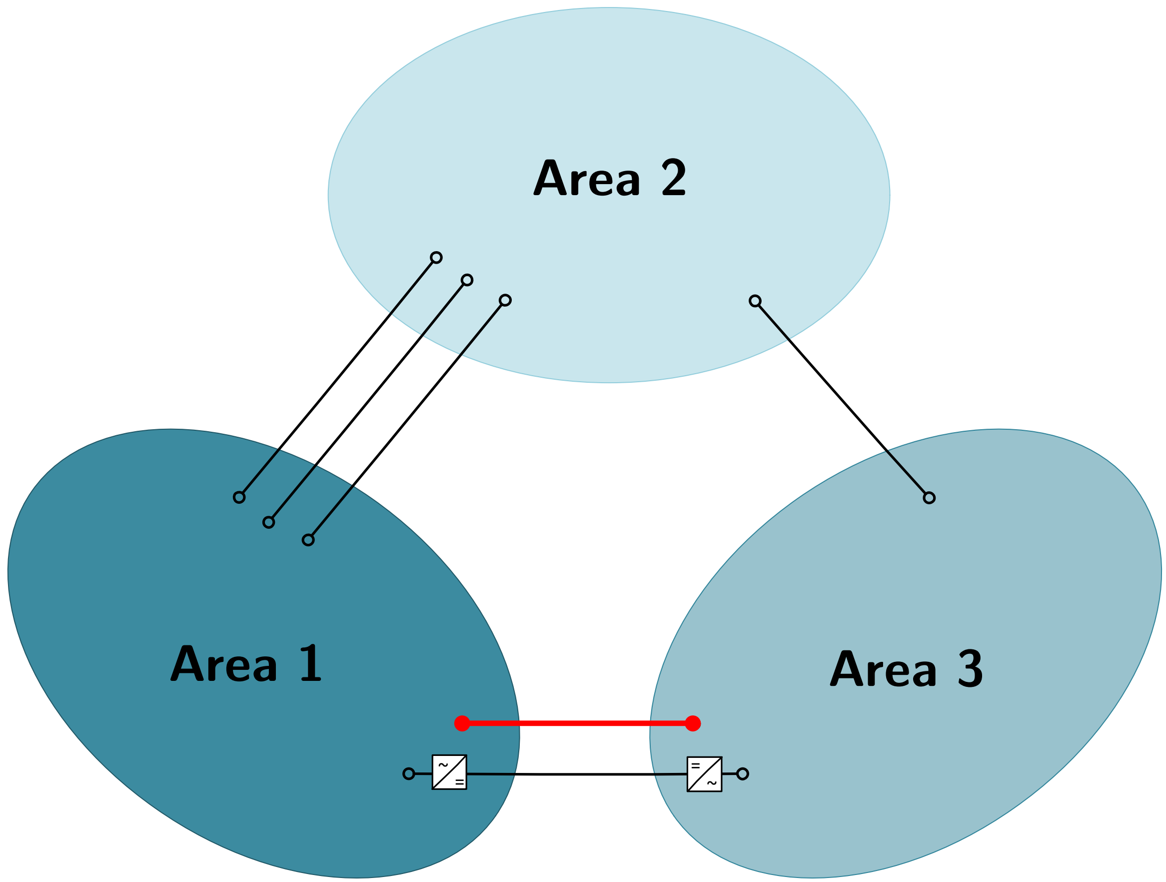
Flow-based market coupling

For **each area**:

- 32 producers
- 17 consumers

Four **different situations**:

- Normal operation
- Limited transmission capacity



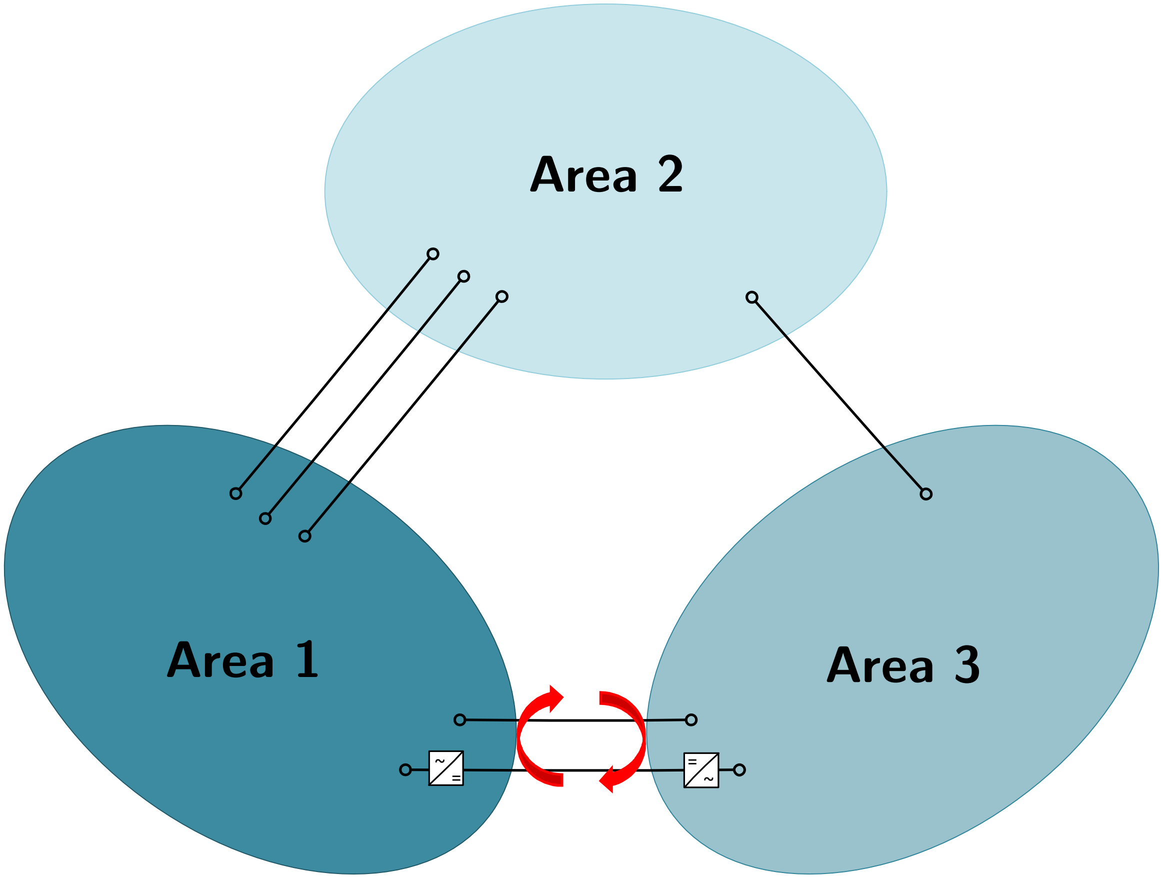
Flow-based market coupling

For **each area**:

- 32 producers
- 17 consumers

Four **different situations**:

- Normal operation
- Limited transmission capacity
- Loop-flows



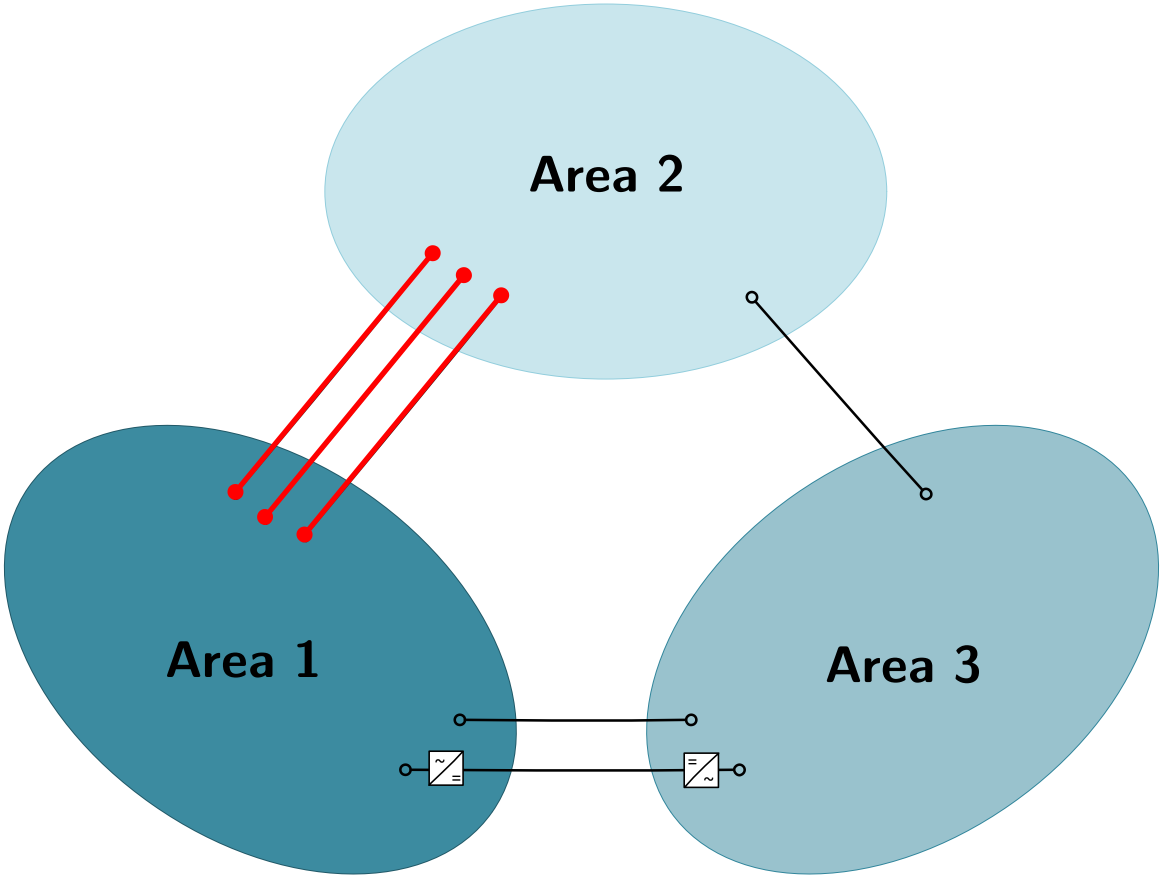
Flow-based market coupling

For **each area**:

- 32 producers
- 17 consumers

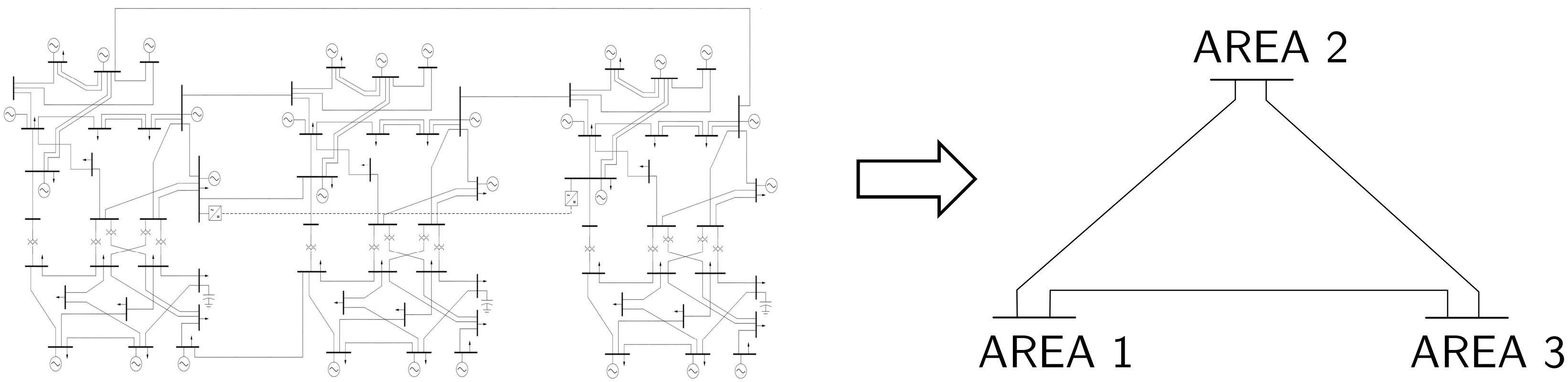
Four **different situations**:

- Normal operation
- Limited transmission capacity
- Loop-flows
- System congestion

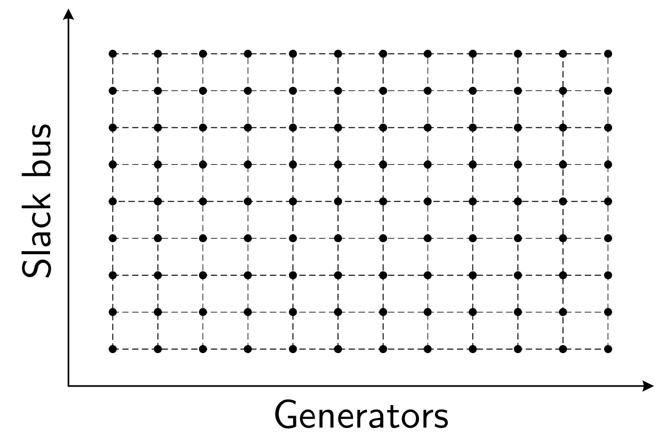


Method

Estimation of the PTDF matrix



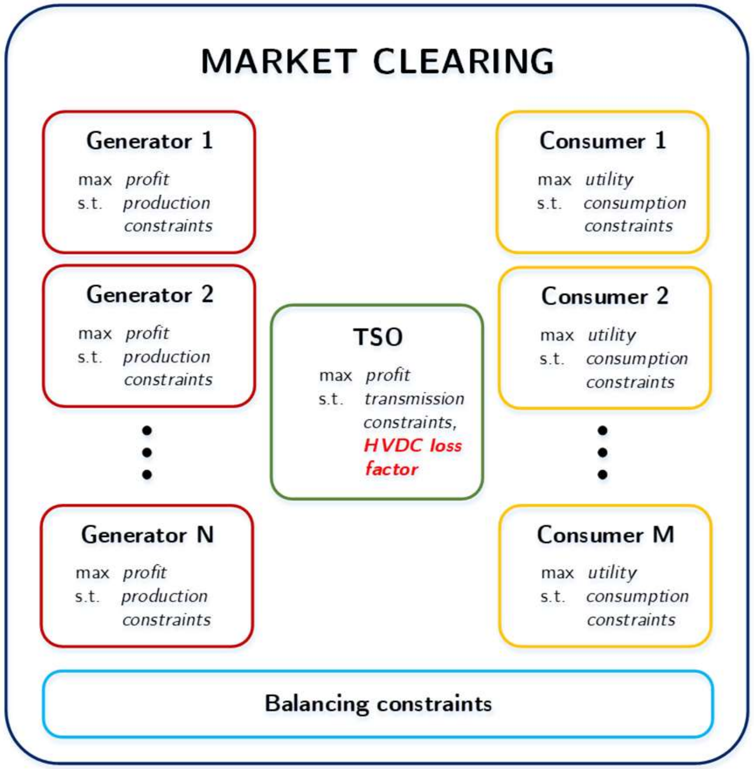
- Marginal variation in one generator at the time
- Selection of the slack bus among the PV buses



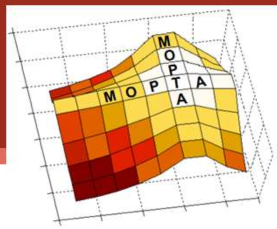
Method

Equilibrium problem with LF

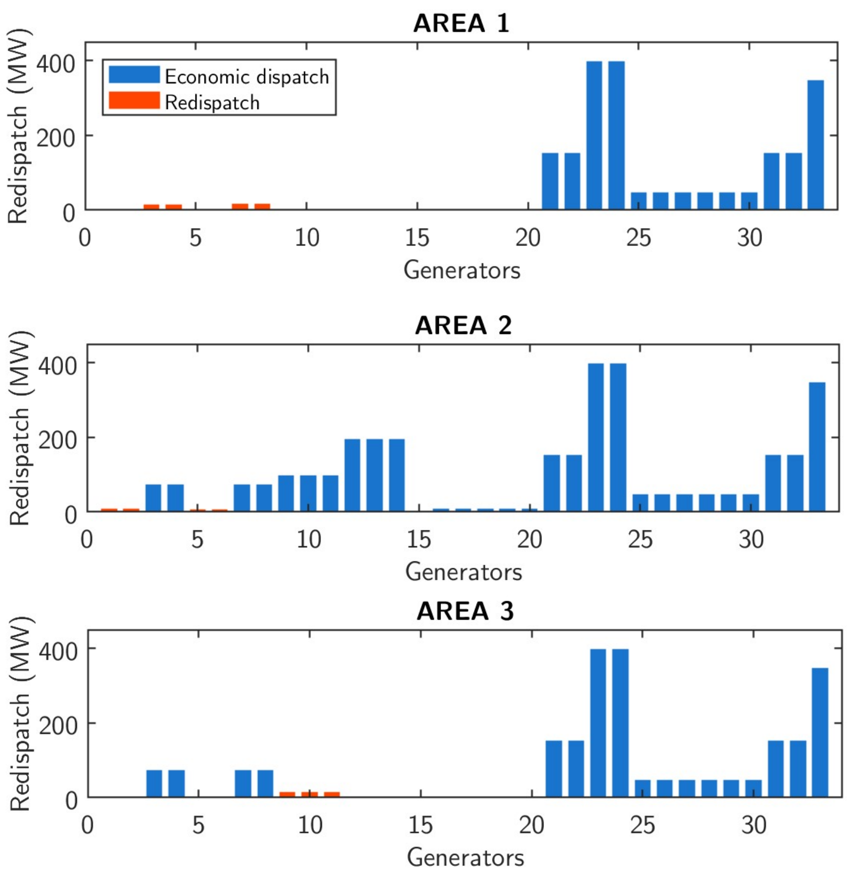
- Each **market participant** seeks to maximize its profit.
- The **loss factor** is introduced in the optimization problem of the TSO.
- The **KKTs** of each problem are derived and the whole problem is solved as an **MCP**.
- The whole problem can be rewritten as an **optimization problem**, where the objective is to maximize the social welfare

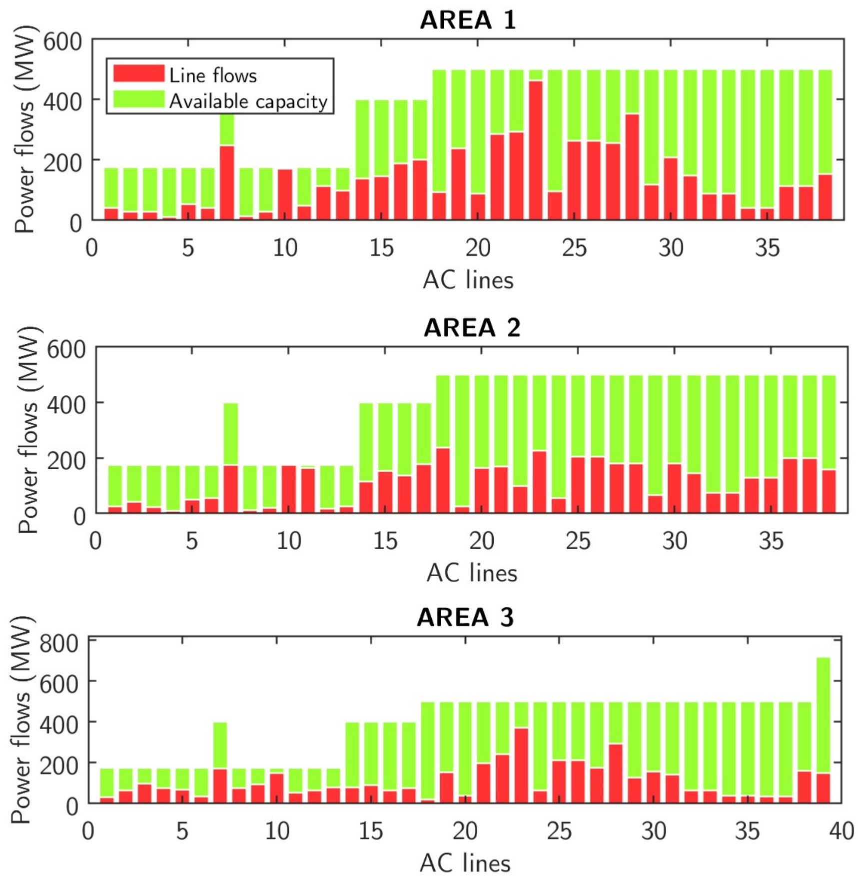


Method



Redispatch





Method

Calculation of the economic benefit

- **Social welfare**

$$SW = PRODUCER\ SURPLUS + CONSUMER\ SURPLUS + CONGESTION\ INCOME$$

- **Cost of losses**

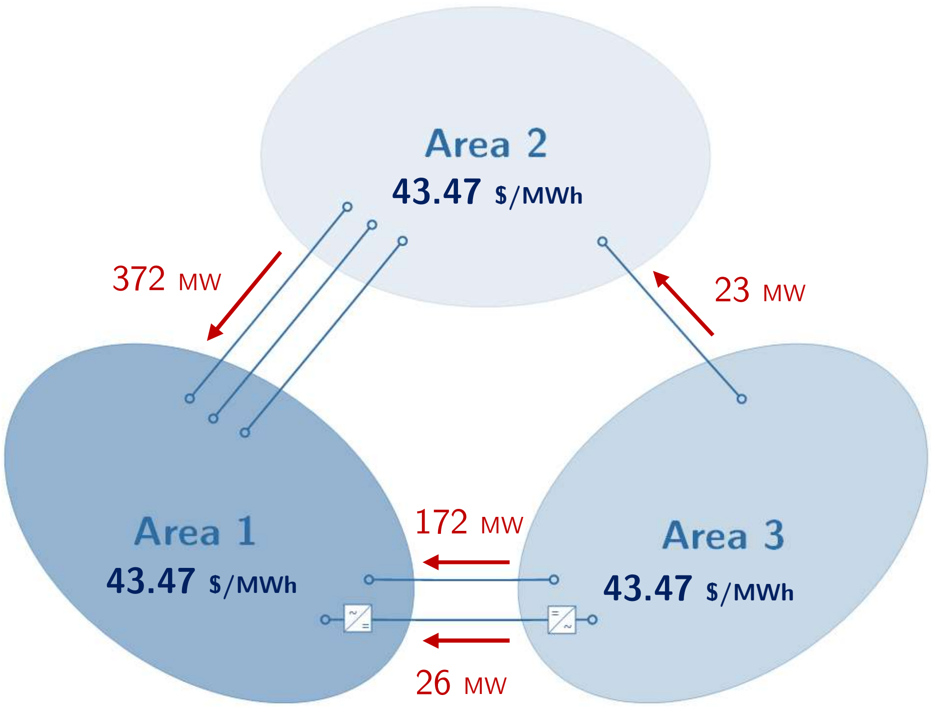
$$COST_{AC} = P_{loss,area} \cdot PRICE_{area}$$
$$COST_{HVDC} = P_{loss,area} \cdot PRICE_{area}$$
$$COST_{HVDC\_LF} = P_{loss,area} \cdot PRICE_{area} - \Delta PRICE_{area} \cdot F_{HVDC}$$

- **Economic benefit**

$$ECONOMIC\ BENEFIT = \Delta SW - \Delta COST_{AC} - \Delta COST_{HVDC}$$

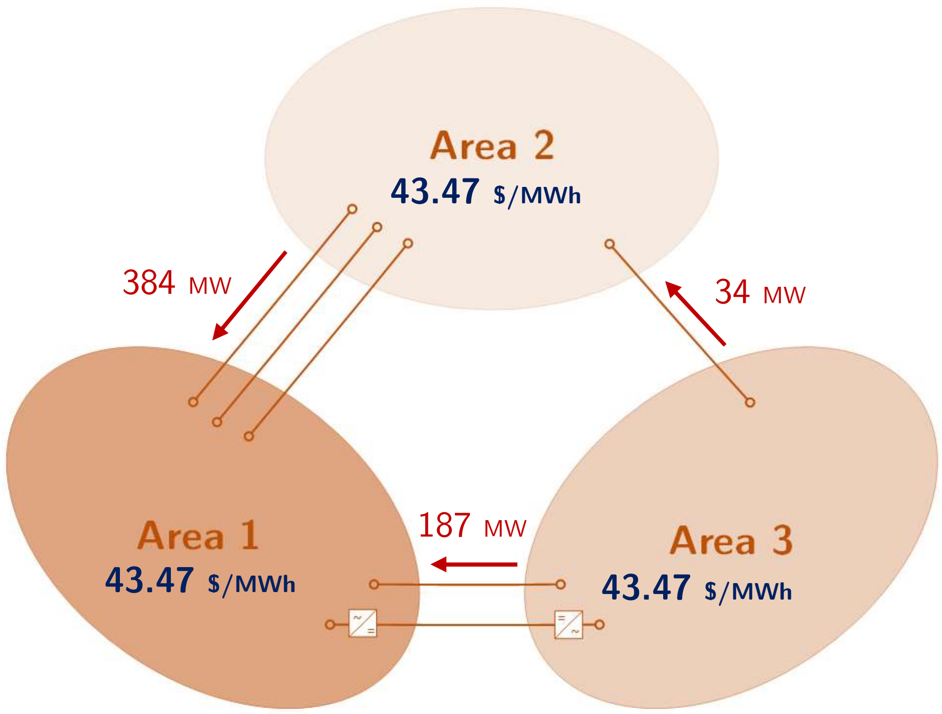
Situation 1: normal operation

NO LOSS FACTOR



SW = 551,533.7 $/h

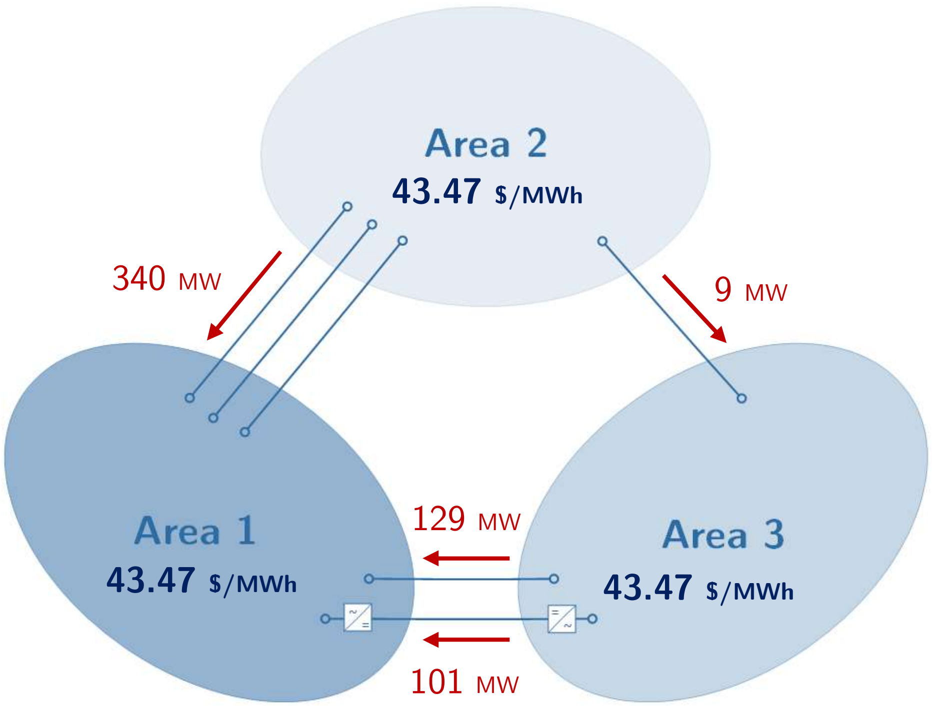
LOSS FACTOR



SW = 551,533.7 $/h

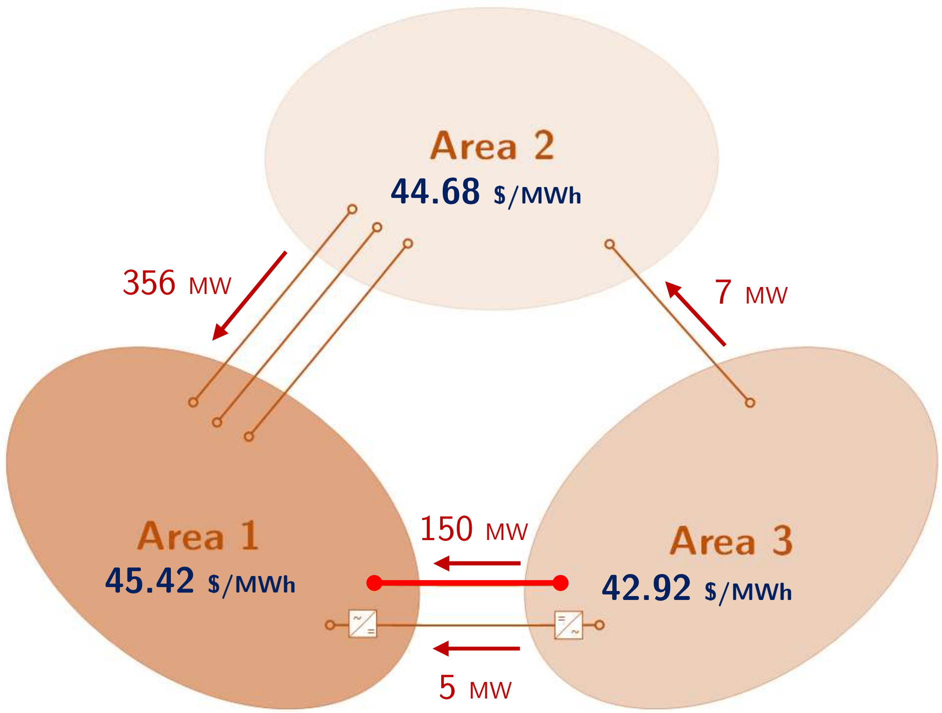
Situation 2: limited capacity

NO LOSS FACTOR



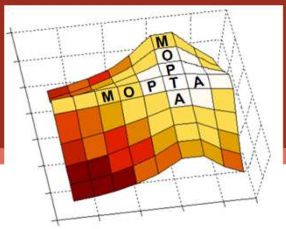
SW = 551,533.7 $/h

LOSS FACTOR

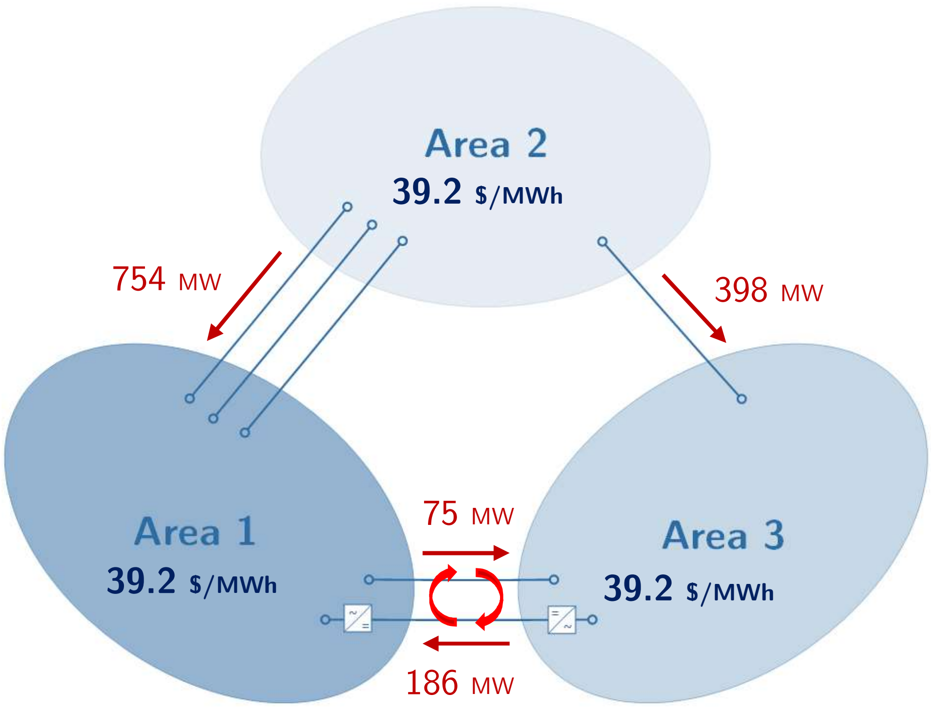


SW = 551,372.5 $/h

Situation 3: loop flow

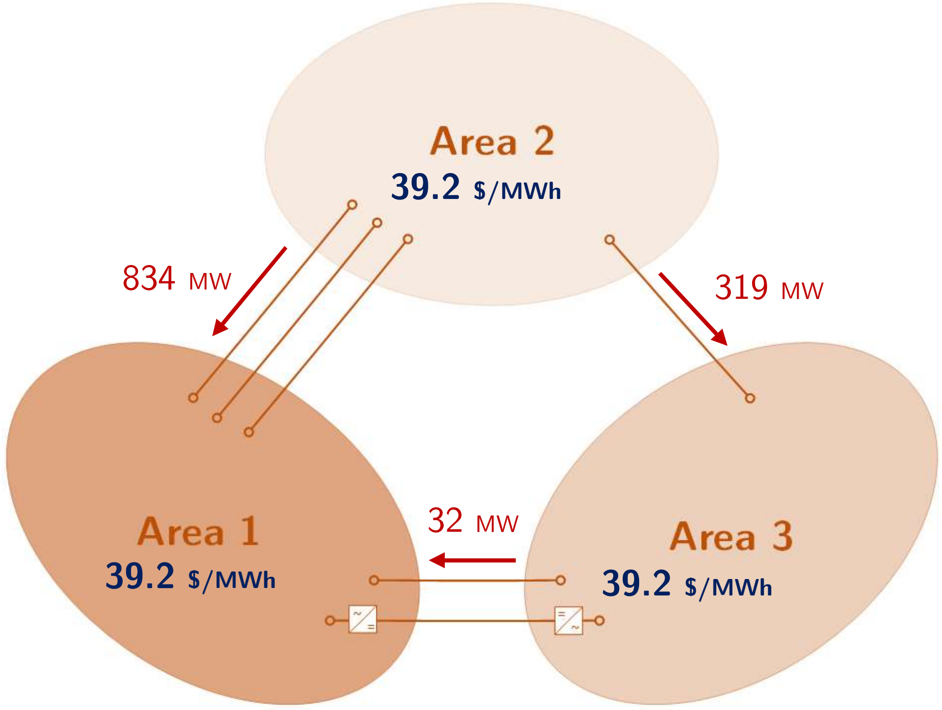


NO LOSS FACTOR



SW = 674,293.1 $/h

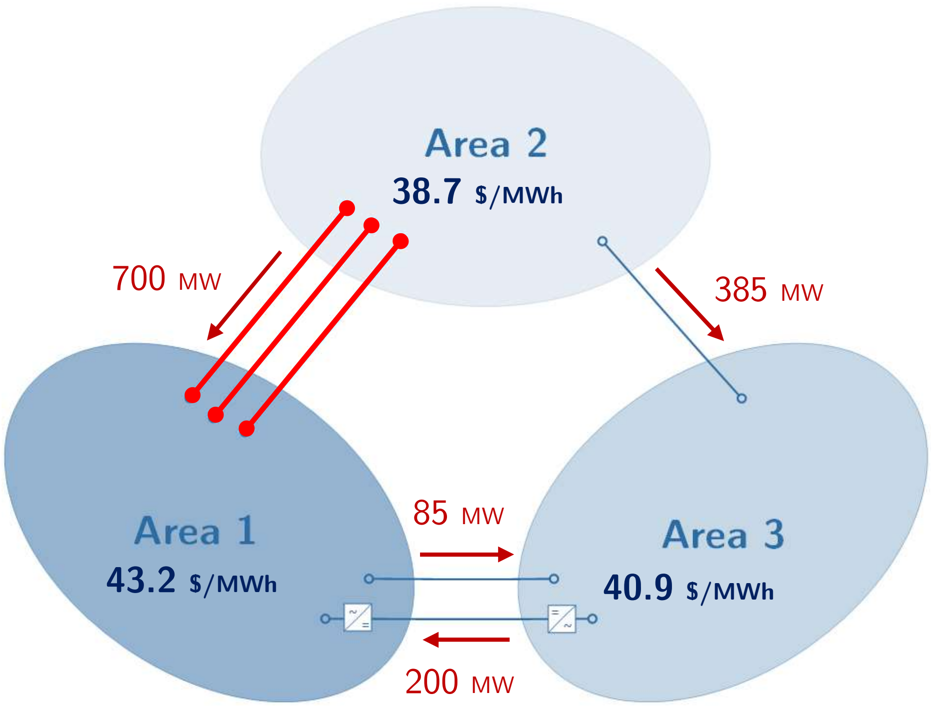
LOSS FACTOR



SW = 674,293.1 $/h

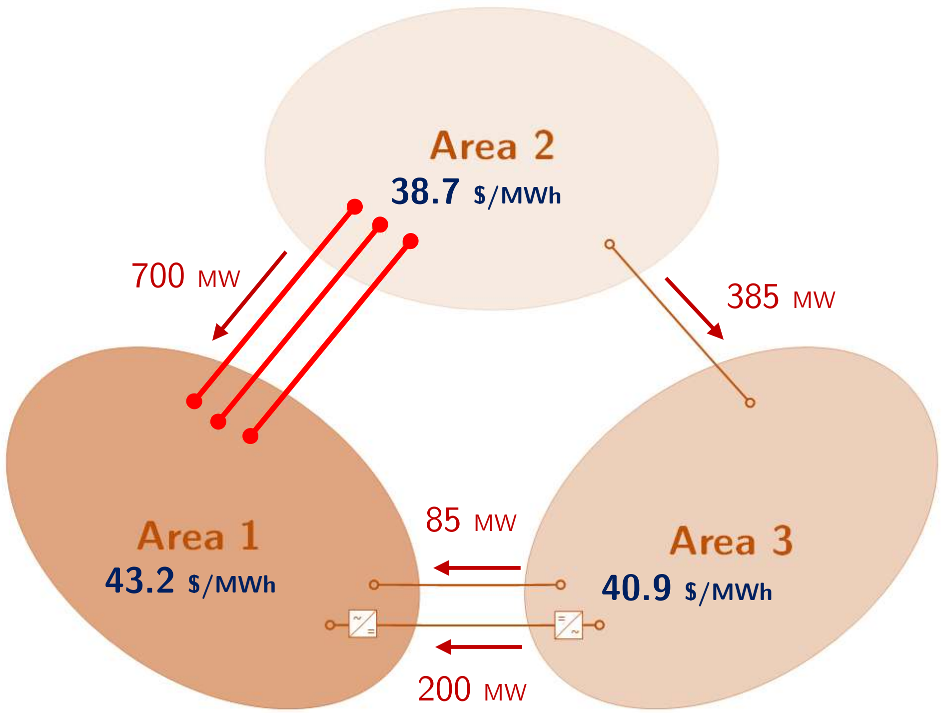
Situation 4: system congestion

NO LOSS FACTOR



SW = 672,277.7 $/h

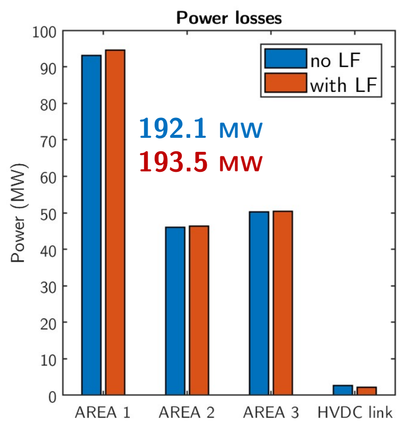
LOSS FACTOR



SW = 672,277.7 $/h

Economic evaluation

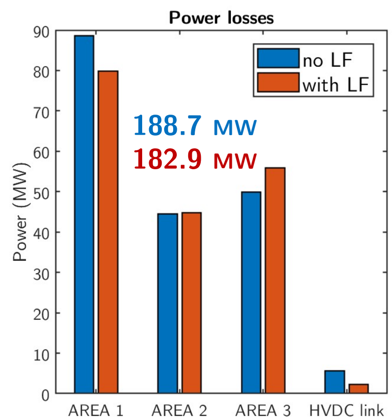
NORMAL OPERATION



- The penalization of the HVDC line results in an increase of losses in the AC system

ECONOMIC LOSS
96.5 $/h

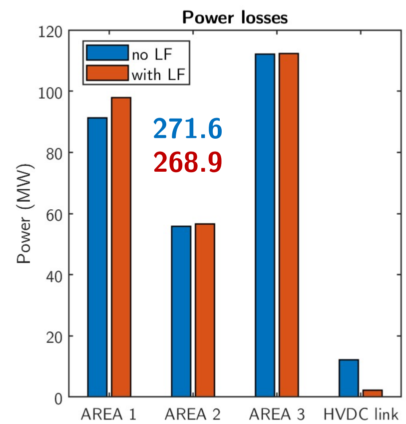
REACHING AC LIMITS



- AC system used up to the maximum capacity
- Different nodal prices due to «congestion»
- Parallel AC/HVDC line implies different prices in all the nodes

ECONOMIC LOSS
88.4 $/h

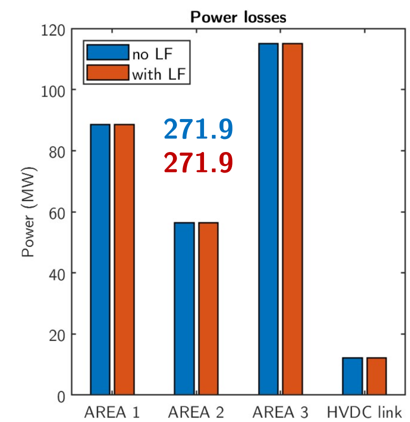
LOOP FLOWS



- With the introduction of the LF loop flows are avoided

ECONOMIC BENEFIT
114.1 $/h

CONGESTION



- Nodal prices differ because of the already existing congestion
- No differences

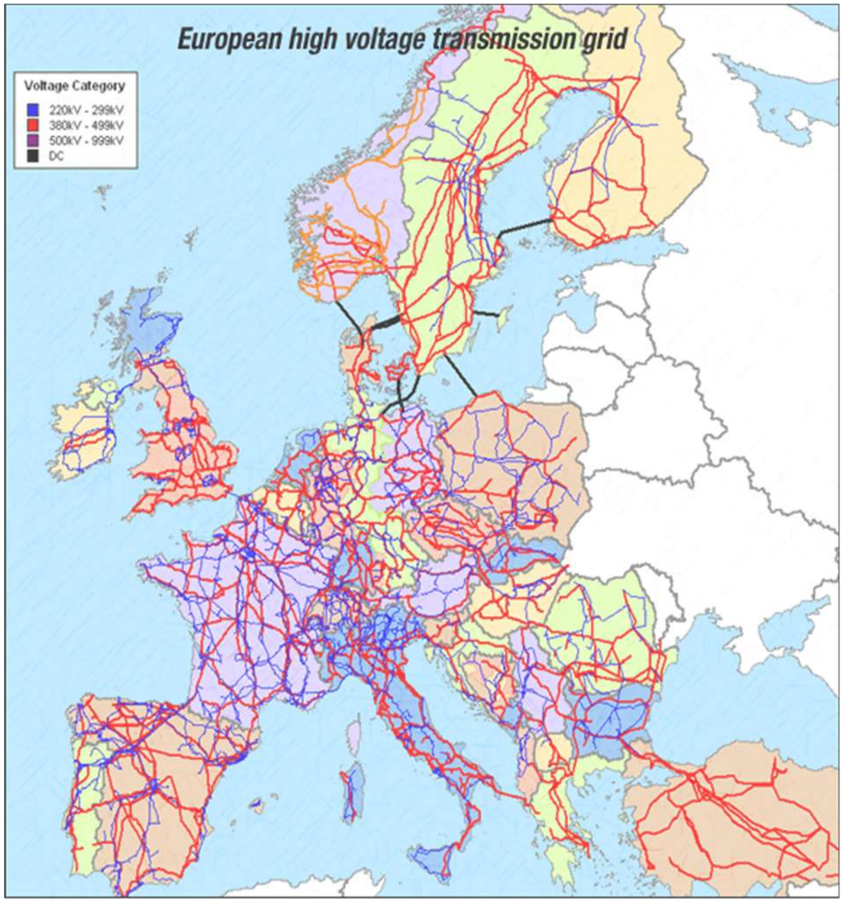
ECONOMIC BENEFIT
0 $/h

Conclusion

- The loss factor can act positively or negatively w.r.t. the amount of system losses depending on the **system under investigation**.

- The introduction of loss factors in **specific markets** has to be analyzed using a model that represent those transmission networks.

- In the future, we might consider the introduction of loss factors for **AC interconnectors**, to balance the usage of the two systems.

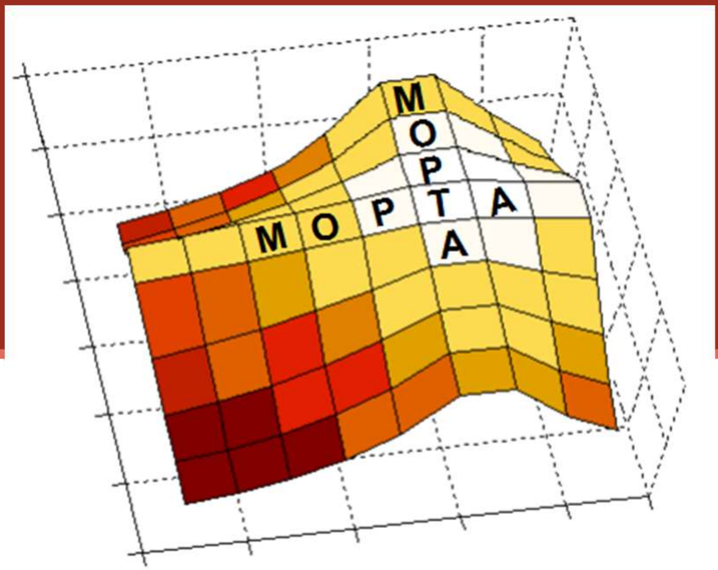


DTU Technical University of Denmark

multiDC

Thanks for the attention!

antosat@elektro.dtu.dk



Lehigh University, Bethlehem PA

August 17, 2018

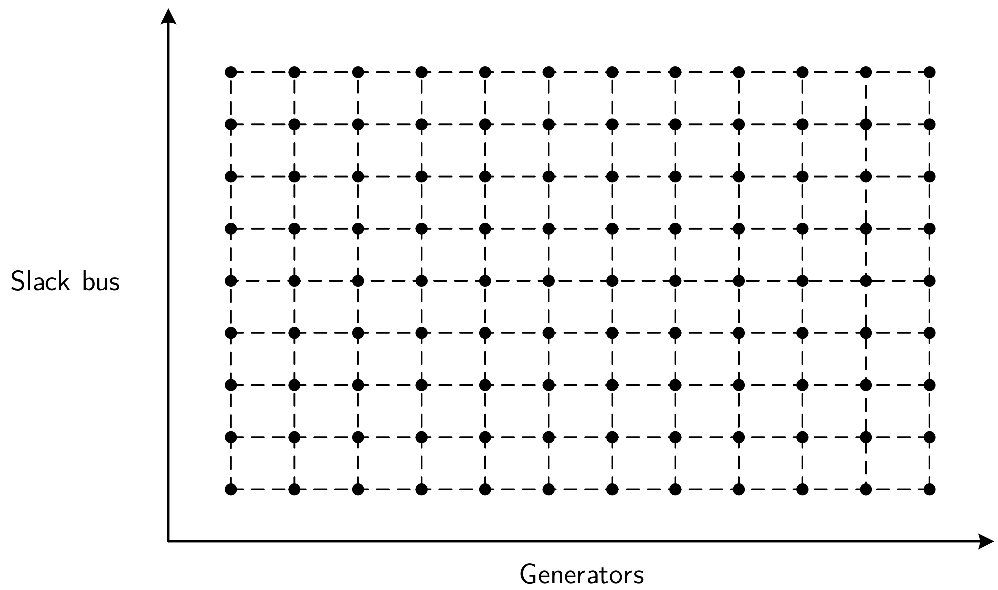
APPENDIX – PTDF matrix

➢ **Marginal variation in one generator at the time**

- Generators output is decreased by 10MW, one at the time
- The average is calculated on a 10-value basis, corrisponding to the number of generators

➢ **Selection of the slack bus among the PV buses**

- The average is calculated on a 11-value basis, corrisponding to the number of PV+slack buses



Generators:

$$\max_{g_i \geq 0} \quad g_i(\lambda_{n:i\in\Psi_n} - c_i)$$
$$\text{s.t.} \quad g_i \leq G_i^{max} \ : \ \gamma_i$$

Consumers:

$$\max_{d_i \geq 0} \quad d_i(u_i - \lambda_{n:i\in\Phi_n})$$
$$\text{s.t.} \quad d_i \leq D_i^{max} \ : \ \zeta_i$$

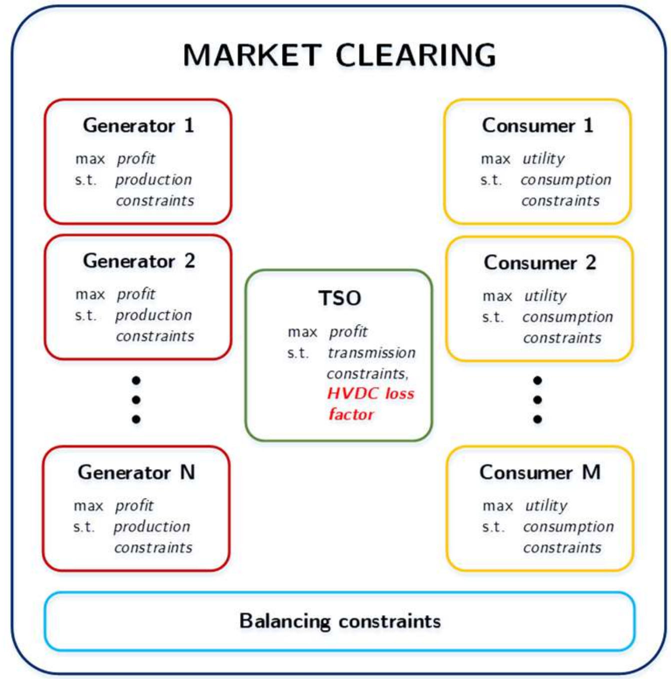
TSO:

$$\max_{\theta^{DA}, f_{DC}^{DA}, p_{los}\ , p_{lossL}} \quad \sum_{n,(m\in\Omega_n)} \lambda_n [B_{nm}(\theta_m - \theta_n)] + \sum_{n,(l\in\Lambda_n)} \lambda_n \mathrm{I}_{nl} f_{DC,l} - \sum_n \lambda_n p_{lossN,n}$$

$$\text{s.t.} \quad B_{nm}(\theta_n - \theta_m) \leq F_{nm}^{max} \ : \ \mu_{ac,nm} \quad \forall n, \forall m \in \Omega_n$$
$$-F_{DC,l}^{max} \leq f_{DC,l} \leq F_{DC,l}^{max} \ : \ \underline{\mu}_{DC,l}, \bar{\mu}_{DC,l} \quad \forall l$$
$$\theta_{slack} = 0 \ : \ \delta$$
$$p_{lossL,l} \geq \alpha_l f_{DC,l} + \beta_l \ : \ \sigma_l^+ \quad \forall l$$
$$p_{lossL,l} \geq \alpha_l(-f_{DC,l}) + \beta_l \ : \ \sigma_l^- \quad \forall l$$
$$p_{lossN,n} = \sum_l M_{nl} \cdot p_{lossL,l} \ : \ \tau_n \quad \forall n$$

Balancing equation:

$$\sum_{i\in\Phi_n} d_i - \sum_{l\in\Lambda_n} \mathrm{I}_{nl} f_{DC,l} + \sum_{m\in\Omega_n} B_{nm}(\theta_n - \theta_m) - \sum_{i\in\Psi_n} g_i + p_{lossN,n} = 0 \ : \ \lambda_n \quad \forall n$$



APPENDIX – Equivalent MCP

Generators:

$$0 \le c_i - \lambda_{n:i\in\Psi_n} + \gamma_i \quad \perp \quad g_i \ge 0$$
$$0 \le G_i^{max} - g_i \quad \perp \quad \gamma_i \ge 0$$

Consumers:

$$0 \le \lambda_{n:i\in\Phi_n} - u_i + \zeta_i \quad \perp \quad d_i \ge 0$$
$$0 \le D_i^{max} - d_i \quad \perp \quad \zeta_i \ge 0$$

TSO:

$$0 = \sum_{m\in\Omega_n} B_{nm}\big(\lambda_n - \lambda_m + \mu_{ac,nm} - \mu_{ac,mn}\big)\ (+\delta), \quad \theta_n \text{ free} \quad \forall n$$
$$0 = -\sum_{n,(l\in\Lambda_n)} \lambda_n \mathrm{I}_{nl} - \underline{\mu}_{DC,l} + \bar{\mu}_{DC,l} + \alpha_l\sigma_l^+ - \alpha_l\sigma_l^-, \quad f_{DC,l} \text{ free} \quad \forall l$$
$$0 = -\sigma_l^+ - \sigma_l^- - \sum_n N_{nl}\cdot\tau_n, \quad p_{lossL,l} \text{ free} \quad \forall l$$
$$0 = \tau_n + \lambda_n, \quad p_{lossN,n} \text{ free} \quad \forall n$$
$$0 = \theta_{slack}, \quad \delta \text{ free}$$
$$0 \le F_{nm}^{max} - B_{nm}(\theta_n - \theta_m) \ \perp\ \mu_{ac,nm} \ge 0 \quad \forall n, \forall m\in\Omega_n$$
$$0 \le F_{DC,l}^{max} + f_{DC,l} \ \perp\ \underline{\mu}_{DC,l} \ge 0 \quad \forall l$$
$$0 \le F_{DC,l}^{max} - f_{DC,l} \ \perp\ \bar{\mu}_{DC,l} \ge 0 \quad \forall l$$
$$0 \le p_{lossL,l} - \alpha_l f_{DC,l} - \beta_l \ \perp\ \sigma_l^+ \ge 0 \quad \forall l$$
$$0 \le p_{lossL,l} + \alpha_l f_{DC,l} - \beta_l \ \perp\ \sigma_l^- \ge 0 \quad \forall l$$
$$0 = p_{lossN,n} - \sum_l M_{nl}\cdot p_{lossL,l}, \quad \tau_n \text{ free} \quad \forall n$$

Balancing equation:

$$0 = \sum_{i\in\Phi_n} d_i - \sum_{l\in\Lambda_n} \mathrm{I}_{nl} f_{DC,l} + \sum_{m\in\Omega_n} B_{nm}(\theta_n - \theta_m) - \sum_{i\in\Psi_n} g_i + p_{lossN,n}, \quad \lambda_n \text{ free} \quad \forall n$$

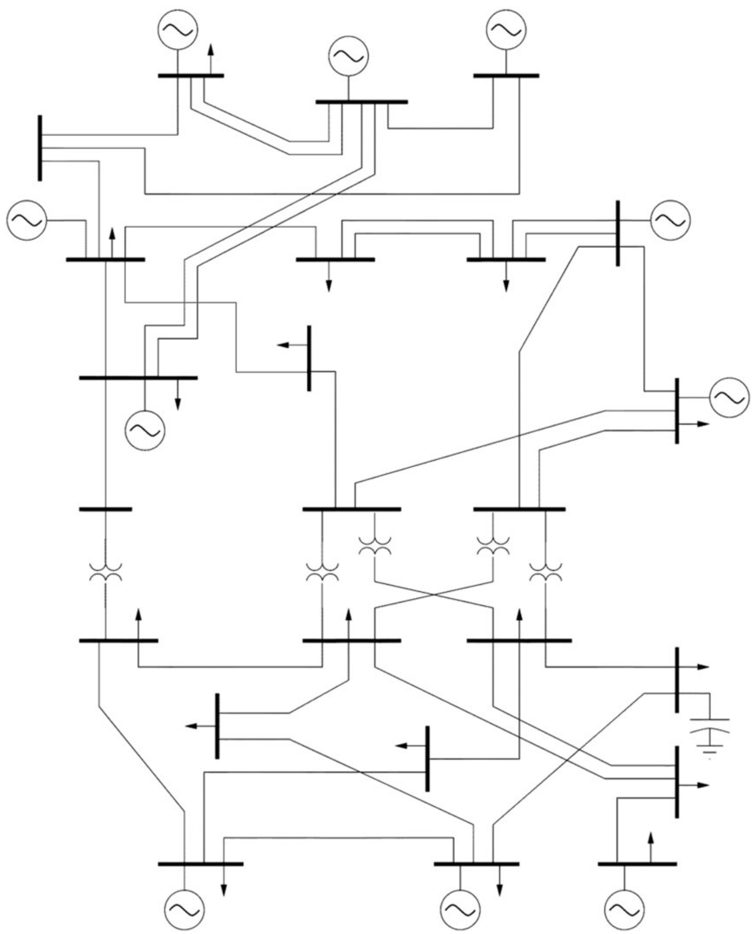
APPENDIX – Optimization problem

$$\max_{\boldsymbol{g},\boldsymbol{d},\boldsymbol{\theta},\boldsymbol{f}_{DC},\boldsymbol{p}_{lossN},\boldsymbol{p}_{lossL}} \quad \sum_j d_j u_j - \sum_i g_i c_i$$

$$\begin{aligned}
\text{s.t.} \quad & 0 \leq \boldsymbol{g} \leq G^{max} \quad : \quad \underline{\boldsymbol{\gamma}}, \overline{\boldsymbol{\gamma}} \\
& 0 \leq \boldsymbol{d} \leq D^{max} \quad : \quad \underline{\boldsymbol{\zeta}}, \overline{\boldsymbol{\zeta}} \\
& -\boldsymbol{F}_{AC}^{max} \leq \boldsymbol{B}_{line}\boldsymbol{\theta} \leq \boldsymbol{F}_{AC}^{max} \quad : \quad \underline{\boldsymbol{\mu}}_{AC}, \overline{\boldsymbol{\mu}}_{AC} \\
& -\boldsymbol{F}_{DC}^{max} \leq \boldsymbol{f}_{DC} \leq \boldsymbol{F}_{DC}^{max} \quad : \quad \underline{\boldsymbol{\mu}}_{DC}, \overline{\boldsymbol{\mu}}_{DC} \\
& \theta_{slack} = 0 \quad : \quad \delta \\
& \boldsymbol{p}_{lossL} \geq \boldsymbol{\alpha}\boldsymbol{f}_{DC,l} + \boldsymbol{\beta} \quad : \quad \boldsymbol{\sigma}^{+} \\
& \boldsymbol{p}_{lossL} \geq \boldsymbol{\alpha}\left(-\boldsymbol{f}_{DC,l}\right) + \boldsymbol{\beta} \quad : \quad \boldsymbol{\sigma}^{-} \\
& \boldsymbol{p}_{lossN} = \mathbf{M}_{DC} \cdot \boldsymbol{p}_{lossL} \quad : \quad \boldsymbol{\tau} \\
& \boldsymbol{d}_{\sum j \in \Phi_n} + \mathbf{I}_{DC}\boldsymbol{f}_{DC} + \boldsymbol{B}_{bus}\boldsymbol{\theta} - \boldsymbol{g}_{\sum i \in \Psi_n} + \boldsymbol{p}_{loss} = 0 \quad : \quad \boldsymbol{\lambda}
\end{aligned}$$

APPENDIX – Redispatch

For each area, check if line constraints are violated and dispatch generators to cover the losses.



For each area:

- $\boldsymbol{P}_g^*$ vector with generators dispatch
- $\boldsymbol{P}_d^*, \boldsymbol{Q}_d^*$ vector with active and reactive demand
- $\boldsymbol{F}^*$ vector with cross-border flows

$$\min_{\boldsymbol{P}_g,\boldsymbol{Q}_g,\boldsymbol{V},\boldsymbol{\theta}} \boldsymbol{c}^{\mathrm{T}}\boldsymbol{P}_g + \left(\boldsymbol{P}_g - \boldsymbol{P}_g^*\right)^2 \quad \text{Minimize the deviation from the dispatch}$$

$$\text{s.t.} \quad -\boldsymbol{F}^* - \boldsymbol{P}_g + \boldsymbol{P}_d^* = real(\,diag(\overline{\boldsymbol{V}})\overline{\boldsymbol{Y}}_{bus}^*\overline{\boldsymbol{V}}^*\,) \quad \text{Active power balance}$$

$$-\boldsymbol{Q}_g + \boldsymbol{Q}_d^* = imag(\,diag(\overline{\boldsymbol{V}})\overline{\boldsymbol{Y}}_{bus}^*\overline{\boldsymbol{V}}^*\,) \quad \text{Reactive power balance}$$

$$\left|\overline{V}_i\overline{\boldsymbol{Y}}_{line,ij,i-row}^*\overline{\boldsymbol{V}}^*\right| \leq S_{ij}^{max} \quad \forall(i,j)$$

$$\left|\overline{V}_j\overline{\boldsymbol{Y}}_{line,ji,j-ro}^*\overline{\boldsymbol{V}}^*\right| \leq S_{ji}^{max} \quad \forall(j,i)$$

Apparent flow

$$\boldsymbol{0} \leq \boldsymbol{P}_g \leq \boldsymbol{P}_g^{max} \quad \text{Gen. active power}$$

$$\boldsymbol{Q}_g^{min} \leq \boldsymbol{Q}_g \leq \boldsymbol{Q}_g^{max} \quad \text{Gen. reactive power}$$

$$\boldsymbol{V}^{min} \leq \boldsymbol{V} \leq \boldsymbol{V}^{max} \quad \text{Voltage magnitude}$$

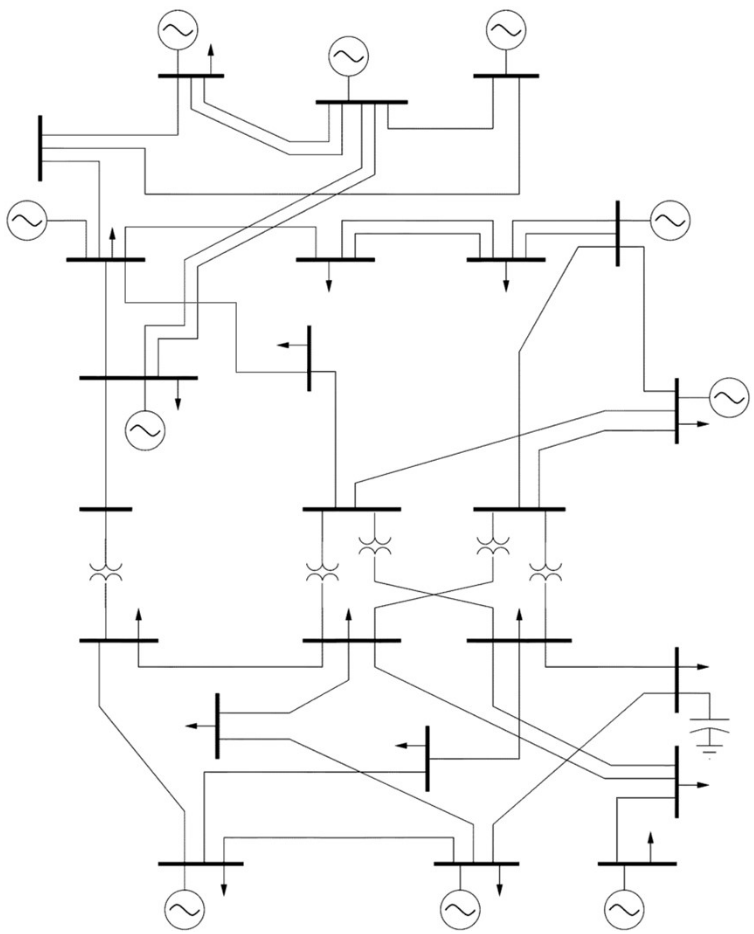
$$\boldsymbol{\theta}^{min} \leq \boldsymbol{\theta} \leq \boldsymbol{\theta}^{max} \quad \text{Voltage angle}$$

$$\theta_{slack} = 0 \quad \text{Slack bus}$$

- Update $\boldsymbol{P}_g^*$

APPENDIX – AC Power flow

Calculate the losses in the AC system and on the HVDC link.



For each area:

- $\boldsymbol{P}_g^*$ vector with generators dispatch
- $\boldsymbol{P}_d^*$ vector with loads
- $\boldsymbol{F}^*$ vector with cross-border flows
- The flows are included as new generators, the corresponding bus is treated as a PV bus with voltage 1 p.u.
- Losses on the interconnectors are calculated as:
$$P_{loss,ij} = R \cdot F_{ij}^2$$
and dived between the two areas.
- Losses on the HVDC link are calculated as:
$$P_{loss,HVDC} = (a_{inv} + a_{rec} + R)F_{ij}^2 + 2bF_{ij} + 2c$$
and divided between the two areas.
- The cost of losses is calculated by multiplying the losses with the price of the corresponding area:
$$COST_{loss} = P_{loss,area} \cdot PRICE_{area}$$