HVDC Line Placement for Maximizing Social Welfare — An Analytical Approach

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***Abstract*—This paper derives a method for HVDC line placement in order to maximize social welfare. Assuming linear generator costs, it shows that for N congestions, there exist exactly $N+1$ marginal generators. It then demonstrates that HVDC lines connecting a high-cost with a low-cost marginal generator lead to the maximization of social welfare. The method poses an upper bound on the line installation costs, which could also be used to increase the efficiency of optimization algorithms. At the same time, it shows that an optimal HVDC placement is not necessarily along the overloaded lines. Furthermore, an algorithm for HVDC placement is developed taking into account the line capacity and the generator constraints to estimate expected generation cost savings. The validity of the approach is demonstrated on two case studies: a 10-bus network, and a simplified European network.**

***Index Terms*—HVDC line, Placement, marginal generators, line congestions, Lagrangian multipliers**

I. Introduction

Aging power system infrastructure and the need for increased integration of fluctuating generation call for substantial investments in transmission infrastructure and enhanced power system flexibility. Several measures are being investigated in order to expand the transmission capacity in the network. High Voltage Direct Current lines (HVDC) belong to the most attractive candidates as they combine additional transfer capacity with controllability in their power flow.

The objective of this paper is to propose a method for HVDC line placement, relying on power system properties. The goal is either to identify directly the "optimal" solution avoiding iterative optimization procedures, or, at least, define appropriate search-space bounds, thus significantly increasing the efficiency of an optimization algorithm.

First, an upper bound on the installation costs of an HVDC line is derived when the line placement aims to maximize social welfare. Section II provides an overview of the analytical approach, while Sections III and IV describe the theoretical derivations supporting the proposed method.

Second, an algorithm for HVDC placement is outlined (see Section V). Its performance is validated with two case studies in Section VI: in the first, a 10-bus network is used to show that the method can also be valid in an AC-OPF context; in the second, the method is applied on a simplified European network, where several congested lines appear. Section VII concludes this paper.

II. Description of the Approach

Maximizing social welfare is synonymous to the relief of existing congestions, as also shown in [1]. Equivalently, the investment objective can be expressed as the elimination of the Lagrangian Multipliers of the line flow inequalities (Line LMs), i.e., to set them to zero. In Section IV, we show that in a DC-OPF context, the Line LMs λ are dependent only on the linear costs of the marginal generators for any given network. Marginal are the generators which are dispatched neither at their minimum nor at their maximum limit. In [2], similar results are obtained for the general case, namely that the nodal prices are a function of the marginal generators costs and the network constraints, including the losses.

Assuming linear generator costs and a DC-OPF context, in Section III, we show that if N lines are congested, there exist exactly $N+1$ marginal generators. Equivalently, eliminating a marginal generator will result in the relief of a line congestion. *Therefore, adding a line with the objective to set the dispatch of the out-of-merit order generator to zero, is equivalent to congestion relief and maximization of the social welfare.* The power produced by this generator can be delivered by the cheaper marginal generator.

Assuming one congested line in our system, there exist exactly two marginal generators[1]: one low-cost generator G_{LC} – with a cost equal to, or lower than, the system marginal cost without congestion – and one high-cost generator G_{HC} – with a cost higher than the system marginal cost. The objective here is to add line capacity, such that the low-cost generator can produce the amount of power initially injected from G_{HC}, i.e. $P'_{\text{LC}} = P_{\text{LC}} + P_{\text{HC}}$. In doing so, the low-cost generator takes on the production share of the high-cost generator. The simplest solution in this case is to add a line, connecting the nodes G_{LC} and G_{HC} with line capacity equal to $C_{DC} = P_{\text{HC}}$. This serves as an upper bound, i.e., any reinforcement which is more expensive than this solution should be discarded (as long as the objective is to relieve the specific congestion). This holds true for the placement of HVDC lines, as the DC technology ensures controllability and the ability to transfer any desired amount of power up to the line's capacity limit. Instead, the power transfer capability of an AC line depends also on its electrical characteristics. It might occur that an AC line with capacity equal to P_{HC} cannot transfer power up to its limit, due to the meshed structure of the network. In this case detailed calculations are necessary in order to determine the optimal characteristics of the AC line. The interested reader can refer to Ref. [3] for upper bounds in loadings of long AC lines.

A DC line connecting the marginal generators with capacity P_{HC} defines the *upper bound* for line installation costs, and sets the benchmark for the "best performance" with respect to social welfare maximization. Depending on the properties of the network, further studies taking installation costs into

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[1]See Section III for a more detailed treatment of the underlying theory.

account can identify the (near-)optimal line placement between nodes with a shorter distance or with capacity less than P_{HC}.

Proposition: In case of one congested line, there exist two marginal generators: G_{LC}, with lower marginal costs, and G_{HC}, with higher marginal costs. If G_{LC} has sufficient capacity, a DC line which directly connects the two marginal generators with line capacity equal to P_{HC} (the power produced from the expensive generator) serves as the upper bound with respect to costs. Any solution which is more expensive than this should be discarded.

The above Proposition also shows that the placement of a DC line which relieves congestion and maximizes social welfare is not necessarily along the congested line, but rather on the path that connects the two marginal generators.

The Proposition refers to one congested line. In case N line congestions exist in the system, at least N additional DC lines will be necessary, assuming that each line addition relieves one congestion. It may occur though, that the low-cost marginal generator does not have enough available capacity in order to take up the total production of the expensive marginal generator. In such a case, either the line will remain congested or a different congested line will appear. As a result, the number of congested lines N defines also the lower bound of the number of DC lines necessary to relieve all congestions.

Concerning a system with several congested lines and marginal generators, the first line should be placed between the most expensive marginal generator and the least-cost marginal generator in the system, assuming that the low-cost generators have enough available power capacity. In Section V we outline an algorithm for the placement of HVDC lines in a sequential manner, taking into account the limitations in the installed capacity of the generators and the thermal capacity of the HVDC line. We then apply this algorithm on a case study.

III. Relationship between Number of Marginal Generators and Line Congestions

In this section we show that for N congested lines in the system, there exist exactly $M = N + 1$ marginal generators. For the derivation, generators are modeled with linear costs, and the problem is assumed feasible, i.e., no load should be curtailed due to e.g., limited transmission capacity. Further, identical parallel lines, and generators with equal costs on the same bus are aggregated in one equivalent line or generator respectively.

A. DC-OPF Formulation

We assume a standard DC-Optimal Power Flow formulation, with linear costs for the generators, and based on the Power Transfer Distribution Factors [4]:

$$min \sum_{i=1}^{N_{P_G}} c_i P_{G,i}, \tag{1}$$

subject to:

$$\sum_{i=1}^{N_{P_G}} P_{G,i} - \sum_{i=1}^{N_{P_L}} P_{L,i} = 0, \tag{2}$$

$$\mathbf{PTDF} \cdot |(\mathbf{P_G} - \mathbf{P_L})| \leq \mathbf{F_L}, \tag{3}$$

$$\mathbf{0} \leq \mathbf{P_G} \leq \mathbf{P_{G,max}}. \tag{4}$$

Eq. 1 minimizes the generation costs, where c_i is the linear cost of each generator. Eq. 2 imposes the total generation to equal total demand, while Eq. 4 keeps the generators within limits. Eq. 3 uses the PTDF expression for the line flows, which constrains them below line limits $F_{L,i}$. Boldface symbols denote vectors or matrices.

B. Lagrangian Function of the DC-OPF Problem

The Lagrangian function of the problem, as formulated in Eq. 1-4, is:

$$\begin{aligned}\mathcal{L}(P_G, \nu, \lambda, \mu) = \sum_{i=1}^{N_{P_G}} c_i P_{G,i} + \nu \cdot \left(\sum_{i=1}^{N_{P_G}} P_{G,i} - \sum_{i=1}^{N_{P_L}} P_{L,i} \right) \\ + \sum_{i=1}^{N_L} \lambda_i^+ \cdot [\mathbf{PTDF_i} \cdot (\mathbf{P_G} - \mathbf{P_L}) - F_{L,i}] \\ + \sum_{i=1}^{N_L} \lambda_i^- \cdot [-\mathbf{PTDF_i} \cdot (\mathbf{P_G} - \mathbf{P_L}) - F_{L,i}] \\ + \sum_{i=1}^{N_{P_G}} \left(\mu_i^+ \cdot (P_{G,i} - P_{G,i,max})\right) + \sum_{i=1}^{N_{P_G}} \left(\mu_i^- \cdot (-P_{G,i})\right)). \end{aligned} \tag{5}$$

Here, ν is the Lagrangian multiplier for the equality constraint; λ is a vector containing the Lagrangian multipliers for the line flow inequalities (λ_i^+ in the defined as positive direction and λ_i^- in the opposite direction); and μ is a vector containing the Lagrangian multipliers for the generation inequality constraints.

C. Derivation of the Relationship

If there is no congestion in the system, the generators should be dispatched according to the merit-order curve. Assuming linear generator costs, which implies constant marginal costs, a cheaper generator should be dispatched to its full capacity, before a more expensive generator is committed. As a result, in case of no congestion, there is only one marginal generator.

Assuming that there is a line congestion in the system, there should exist at least two marginal generators[2]. According to the Karush-Kuhn-Tucker (KKT) conditions, $\partial\mathcal{L}/\partial P_G = 0$. Additionally, $\mu^+ = \mu^- = 0$ for all marginal generators, and $\lambda^+ = \lambda^- = 0$ for all non-congested lines (for more information on the derivation of the KKT conditions in a general optimization problem see, e.g., [5]). Without loss of generality, we assume that the single congested line k has a line flow in the positive direction, i.e., $\lambda_k^+ \neq 0$ and $\lambda_k^- = 0$. Let the marginal generators be denoted as P_{Gm} and P_{Gn}. Then:

$$c_m + \nu + \lambda_k^+ \cdot PTDF_{k,m} = 0, \tag{6}$$

$$c_n + \nu + \lambda_k^+ \cdot PTDF_{k,n} = 0. \tag{7}$$

Eq. 6 and Eq. 7 describe a 2x2 linear system, with two equations and two unknowns: ν and λ_k^+. This yields a unique solution. If a third marginal generator P_{Gp} existed for a single congestion, this would have resulted in a 3x2 linear system. In order for such a system to have a solution, the third equation should be linearly dependent on the first two. This implies that the linear cost c_p and the $PTDF_{k,p}$, which describes the

[2]Here, we will not study the boundary condition, where the load demand downstream the congestion is equal to the power flow which results in a line loaded at exactly 100%, without the need to dispatch additional generators (e.g., 100 MW load on a remote bus, which is connected with a 100 MW line to the rest of the system).

effect of power injection to bus p on line k, should obey a certain relationship, as for example:

$$c_p = c_m + \frac{c_m - c_n}{PTDF_{k,n} - PTDF_{k,m}} \cdot (PTDF_{k,m} - PTDF_{k,p}) \,. \quad (8)$$

Such a relationship between the generator costs and $PTDF$ factors is very difficult to occur in reality. Even under this extreme condition though, it has been found through simulations that the optimization will often converge to a solution dispatching one of the generators to its upper or lower bound and let the rest as marginal.

Concluding this section, it has been shown that for linear generator costs and N congested lines, there exist almost always $M = N + 1$ marginal generators. A case where $M > N+1$ could only exist in a case, which is highly improbable to occur in most real power systems. Still, it has been observed that even under this condition, the optimization will usually converge to a solution where there exist $M = N + 1$ marginal generators.

IV. Calculation of the Lagrangian Multiplier of a Congested Line

We rewrite Eq. 6 and Eq. 7, adding the factor $\xi_k \in \{1, -1\}$, so that the equations are independent of the flow direction the line k is congested:

$$c_m + \nu + \xi_k \cdot \lambda_k \cdot PTDF_{k,m} = 0, \quad (9)$$

$$c_n + \nu + \xi_k \cdot \lambda_k \cdot PTDF_{k,n} = 0. \quad (10)$$

In case of one congested line, the Lagrangian multiplier of the line can be computed from the following relationship:

$$\lambda_k = \xi_k \cdot \frac{c_m - c_n}{PTDF_{k,n} - PTDF_{k,m}}, \quad (11)$$

where c_m and c_n are the linear costs of the marginal generators; and $PTDF_{k,m}$ and $PTDF_{k,n}$ are the PTDFs of the congested line k for the nodes m, n, where the marginal generators are connected.

If there is more than one congested line, the line Lagrangian multipliers can be derived by solving a linear system. For example, for two congested lines, a 3x3 linear system must be solved:

$$\begin{bmatrix} 1 & \xi_{k1} \cdot PTDF_{k1,g1} & \xi_{k2} \cdot PTDF_{k2,g1} \\ 1 & \xi_{k1} \cdot PTDF_{k1,g2} & \xi_{k2} \cdot PTDF_{k2,g2} \\ 1 & \xi_{k1} \cdot PTDF_{k1,g3} & \xi_{k2} \cdot PTDF_{k2,g3} \end{bmatrix} \cdot \begin{bmatrix} \nu \\ \lambda_{k1} \\ \lambda_{k2} \end{bmatrix} = - \begin{bmatrix} c_{g1} \\ c_{g2} \\ c_{g3} \end{bmatrix} \quad (12)$$

Here, $\xi_{kj} \in \{1, -1\}$. If the congested line has a flow along the defined positive direction, then $\xi_{kj} = 1$. If the power flows along the opposite direction, then $\xi_{kj} = -1$. The unknown variable ν is the Lagrangian multiplier for the equality constraint $\sum P_G - \sum P_L = 0$. The value of ν is also the nodal price at the slack bus.

Concluding this section, it becomes apparent that for a given network the Line LMs are only dependent on the costs of the marginal generators. By eliminating the marginal generators, line congestions are relieved and, thus, the maximization of the social welfare is achieved.

V. Algorithm for HVDC Placement

In this section, an algorithm is proposed for sequential placement of HVDC lines in a system. For a given HVDC capacity, the algorithm identifies the placement which will lead to the highest cost savings. To maximize cost savings, except for the marginal costs of the generators, equally important is the amount of power that can be shifted between the marginal generators. This is taken into account in Eq. 13 and Eq. 14, which, for a given HVDC capacity C_{DC}, identify the high-cost (G_{HC}) and low-cost (G_{LC}) marginal generators that the line should connect. $P_{G,i}$ is the active power output and $P_{G,i,max}$ is the maximum active power limit of Generator i.

$$G_{\text{HC}} : \max_i \left(c_i \cdot \min\left\{C_{DC}, P_{G,i}\right\}\right), \quad (13)$$

$$G_{\text{LC}} : \min_i \left(c_i \cdot \min\left\{C_{DC}, (P_{G,i,max} - P_{G,i})\right\}\right). \quad (14)$$

Eq. 15 derives a lower bound on the cost savings that can be achieved through the line placement. This not only provides an estimate on the minimum expected benefit without the need to run an additional DC-OPF, but can also serve as an index for selecting the "optimal" capacity of the line. Eq. 15 is an equality when $C_{DC} = \min\left\{C_{DC}, P_{G,\text{HC}}, (P_{G,\text{LC},max} - P_{G,\text{LC}})\right\}$.

$$\text{CS} \geq (c_{\text{HC}} - c_{\text{LC}}) \cdot \min\left\{C_{DC}, P_{G,\text{HC}}, (P_{G,\text{LC},max} - P_{G,\text{LC}})\right\} \quad (15)$$

In the following, we outline the steps of the proposed algorithm:

1) Run DC-OPF and find marginal generators.
2) Select HVDC Capacity C_{DC}
3) Find the high-cost marginal generator (see Eq. 13).
4) Find the low-cost marginal generator (see Eq. 14).
5) Connect the two generator nodes with an HVDC line.
6) Calculate an estimate for the cost savings (see Eq. 15).

The algorithm can be extended by two additional steps, in order to identify the "optimal" line capacity and placement, with respect to the installation costs. The inclusion of the following steps in case studies remains the object of future work.

- Change the capacity of the line and go to Step 3. Reiterate for the set of available line capacities. Select the line capacity that leads to the maximum lower bound for the cost savings.
- Include the installation costs and calculate the cost savings for all possible pairs of marginal generators. Identify the optimal placing with respect to the maximum lower bound for the cost savings.

After adding an HVDC line between nodes i and j, the DC-OPF described in Eq. 1-4 should be amended with the following additional constraints:

$$P_{DC,i} + P_{DC,j} = 0, \quad (16)$$

$$-P_{DC,max} \leq P_{DC,i} \leq P_{DC,max}, \quad (17)$$

$$-P_{DC,max} \leq P_{DC,j} \leq P_{DC,max}. \quad (18)$$

VI. Case Studies

A. AC-OPF on a 10-bus Network

Figure 1 presents a 10-bus network used to test the validity of the proposed approach. A brief description of the system, as well as the system data can be found on [6][3].

[3]For this case study, all line limits were decreased by 30% in comparison with [6], so as the AC-OPF solution to result to a congested line.

Although the relationships derived in the previous sections assume a DC-OPF context, in this example we will employ a full AC-OPF with quadratic generator costs. The goal here is to demonstrate that the placement method identified through the linear relationships will remain "optimal" even with a more realistic representation of the system, where thermal losses and reactive power flows are taken into account. The AC-OPF results show that line 2-10 is congested, while the marginal generators are located on buses 3 and 8. As shown in Table I, Generator 8 is the out-of-merit generator, having higher costs than Generator 3. According to our approach, the HVDC line should connect nodes 3 and 8 with a capacity higher than 1'532 MW, in order to relieve the congested line.

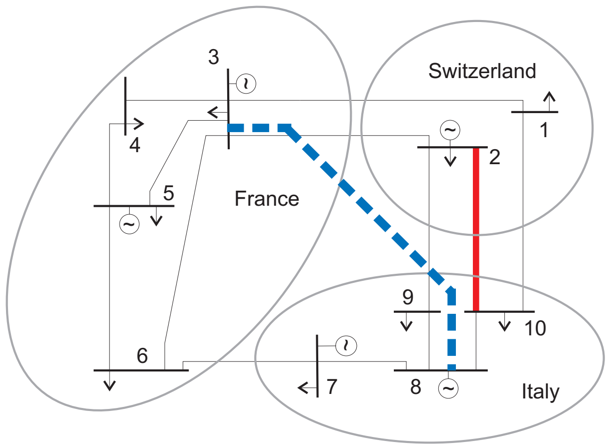

Fig. 1: 10-bus test system. Line 2-10 is congested resulting in two marginal generators on buses 3 and 8. Generator 8 is the out-of-merit order dispatched generator.

TABLE I: Marginal Generators – 10-bus system

Bus	$P_{G,i}$ (MW)	$P_{G,max}$ (MW)	c_i (€/MWh)	$c_{i,quad}$ (€/MWh^2)
3	6'129	8000	24.3	0.00040
8	1'532	2000	50.0	0.00150

Subsequently, we added a single HVDC line with 2'000 MW capacity between every possible pair of buses, and ran the OPF for each case. In total, 45 different HVDC placements have been tested. Figure 2 presents the cost savings achieved with each placement. As it can be observed, an HVDC line between the nodes 4-5 or 4-9 has almost no effect on the dispatch costs. On the other hand, the maximum cost savings are achieved when the line is placed between nodes 3 and 8, exactly where the two marginal generators are connected. In this case, the line congestion is relieved and generator 8 is not dispatched. It should also be noted that placing the HVDC line between nodes 2-10, in parallel with the congested line, results in savings of 7.46%, while the "optimal" placement achieves savings of 10.82%. This highlights the fact that the optimal placement of HVDC lines is not necessarily along the overloaded lines.

B. European Network – single-node per country system

A single-node per country European model, as illustrated in Fig. 3, is used in order to test the approach proposed in this paper. The model comprises 32 nodes and 56 interconnecting

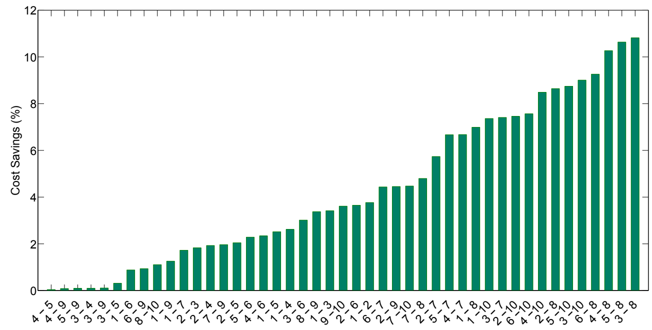

Fig. 2: Cost savings after adding an HVDC line between all possible pair of nodes in the 10-bus system. Maximum cost savings are achieved when the HVDC line connects the two marginal generators (buses 3 and 8).

lines. The line data are aggregations of real data provided by UCTE (now ENTSO-E) (www.entsoe.eu). The generation and load data are taken from the RES 2050 Scenario, as described in Ref. [7]. For this snapshot, 70 generators are assumed in total. On each node, at least two different generators are connected, representing aggregations of RES (Renewable Energy Sources) and conventional power plants respectively. An additional generator is assumed for countries operating nuclear power plants. RES comprises wind, solar and hydro, while, the "conventional" power plant includes up to 10 different generation types (e.g., gas-CCGT, gas-CHP, hard coal, lignite, oil, etc.).

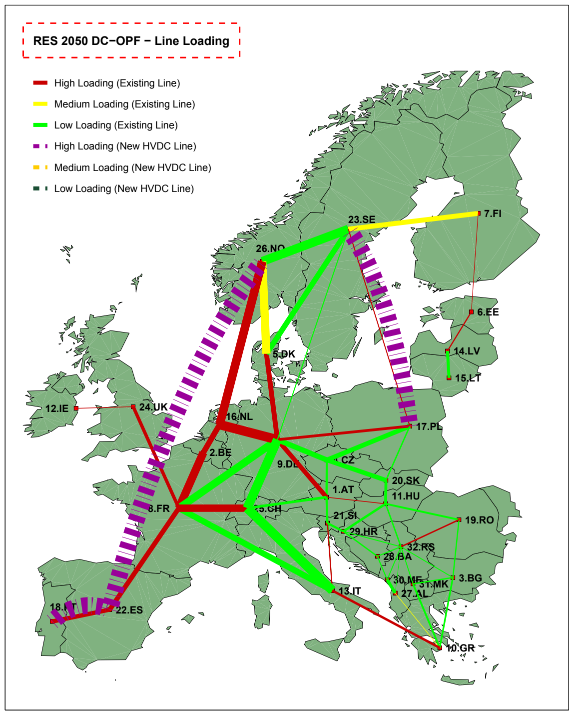

Fig. 3: Single-node per country European model. Line loadings for the RES 2050 scenario after the addition of three HVDC lines. The line thickness is proportional to the transfer capacity of the interconnections.

The maximum capacity $P_{G,max}$ for the RES generators corresponds to the available RES energy for the specific hour and varies throughout the year. The maximum capacity

$P_{G,max}$ for the nuclear and the conventional generators varies based on the availability of each plant. For each country, an average availability of 80% was assumed throughout the year (60% during the six months with the lowest demand, and 100% during the rest of the year).

RES marginal costs are assumed at 0.01 €/MWh, while nuclear power plants have marginal costs of 0.54-0.55 €/MWh. For the conventional generator in each country, the linear costs of each available technology are aggregated in a piece-wise linear function, which is then approximated with an appropriate – "fitted" – quadratic function. For the scope of this work, we will only use the linear coefficient of this quadratic fit. For the RES 2050 generation scenario, the conventional generation costs are comparatively higher, as a carbon tax is projected to be imposed. Differences in fuel costs between summer and winter have also been taken into account.

Table II presents the marginal generators for the system in the base case. In total, there are 19 congested lines and 20 marginal generators, with a total generation dispatch cost of 15'174'361 €/h.

Following the algorithm steps outlined in Section V, we assume an HVDC capacity of 10'000 MW for each of the transmission expansions we carry out in this case study. Such a transmission capacity implies two to five parallel transmission lines, depending on the selected HVDC technology and DC voltage. Applying Eq. 13 and Eq. 14 on the data of Table II, we find that there are two equally good candidates for placing the first HVDC line. These are PT-ES and NO-ES. As the line PT-ES is shorter than the line NO-ES, we prefer at this step to install the line between the nodes PT and ES.

TABLE II: Marginal Generators – No HVDC Expansion.

Bus	$P_{G,i}$ (MW)	$P_{G,max}$ (MW)[a]	c_i (€/MWh)[a]
DK	6'164	7'524	0.01
IE	7'232	10'734	0.01
PT	16'210	30'642	0.01
RO	9'042	10'836	0.01
NO	33'718	44'061	0.01
FR	52'943	54'276	0.54
SE	1'526	11'453	0.55
SI	652	1'059	52.55
AT	1'723	3'952	54.28
NL	8'272	29'941	57.32
LT	422	2'338	67.02
BE	5'386	27'657	92.17
EE	646	5'039	95.02
IT	27'442	45'769	97.05
DE	9'271	111'793	105.86
UK	23'198	75'023	126.71
HU	828	9'298	129.05
PL	20'832	52'763	142.51
ES	20'662	43'408	151.00
GR	4'632	9'336	151.07

[a] The values $P_{G,max}$ and c_i are dependent on the specific snapshot. Please refer to the text for further details.

Fig. 4 presents the achieved cost savings after installing a single HVDC line between all possible nodes in the system. For the 32 nodes in the system, we ran 496 instances of the DC-OPF. For the sake of readability, Fig. 4 presents the three line placements with the lowest and the twenty line placements with the best performance. As it can be observed, placing an HVDC line either between the nodes ES-NO or PT-ES results in equal cost savings of 9.95%.

It is also interesting to note here that by applying Eq. 15, and as $C_{DC} = \min\{C_{DC}, P_{G,\text{HC}}, (P_{G,\text{LC},max} - P_{G,\text{LC}})\}$, we arrive at the same result with respect to the cost savings, without the need to run an additional DC-OPF.

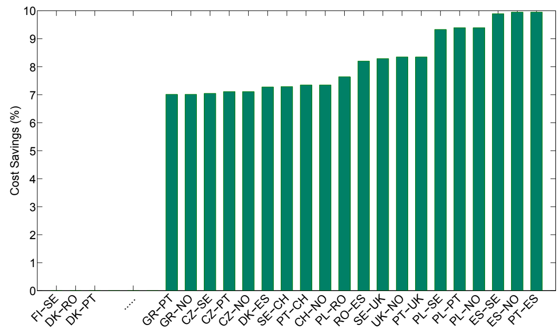

Fig. 4: Placement of first HVDC line. Cost savings after adding an HVDC line between all possible pairs of nodes. From a total of 496 combinations, illustrated are the three pairs with the lowest and the 20 pairs with the best performance.

Table III presents the marginal generators after the addition of an HVDC line between nodes PT-ES. Since in the first placement $C_{DC} < P_{G,HC}$, no congested line was relieved. Still, the added transmission capacity led to the dispatch of lower-cost generators, and thus resulted to a cost reduction of 9.95%. Again, applying Eq. 13 and Eq. 14 on the data of Table III, it seems that the "optimal" HVDC placement would be between the nodes NO-ES. Indeed, running again a DC-OPF for all possible pairs of nodes, Fig. 5 shows that the line placement resulting in the highest cost savings is between nodes NO-ES. Having installed line PT-ES and adding the HVDC line NO-ES results in cost savings of 19.90% in comparison to the base case. The second best option for the second line placement, line ES-SE, results in savings of 19.84% with respect to the base case.

TABLE III: Marginal Generators – HVDC between PT-ES.

Bus	$P_{G,i}$ (MW)	$P_{G,max}$ (MW)[a]	c_i (€/MWh)[a]
DK	6'164	7'524	0.01
IE	7'232	10'734	0.01
PT	26'210	30'642	0.01
RO	9'042	10'836	0.01
NO	33'718	44'061	0.01
FR	52'943	54'276	0.54
SE	1'526	11'453	0.55
SI	652	1'059	52.55
AT	1'723	3'952	54.28
NL	8'272	29'941	57.32
LT	422	2'338	67.02
BE	5'386	27'657	92.17
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UK	23'198	75'023	126.71
HU	828	9'298	129.05
PL	20'832	52'763	142.51
ES	10'662	43'408	151.00
GR	4'632	9'336	151.07

[a] The values $P_{G,max}$ and c_i are dependent on the specific snapshot. Please refer to the text for further details.

The third line placement follows the same procedure. Due to space limitations, we do not present the marginal generators after the second line placement. Still, from Table III, it can be easily observed that after adding a line between NO-ES, the best placement for an additional line with 10'000 MW capacity would be between the nodes PL-SE. As shown in

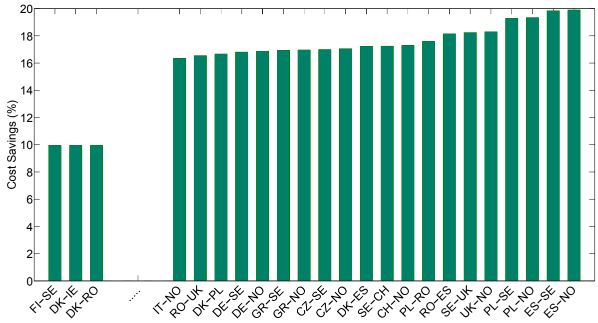

Fig. 5: Cost savings after placing the second HVDC line; first placed between PT-ES.

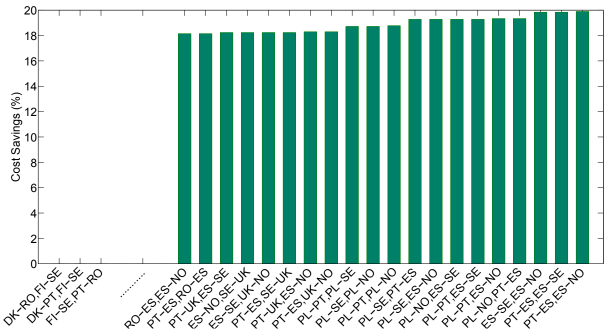

Fig. 7: Cost savings after simultaneous placing of two HVDC lines. From 122’760 combinations in total, shown are the three worst and the 20 with the best performance.

Fig. 6, placing the line between the nodes PL-SE results indeed to the highest cost savings (29.23% in comparison to the base case).

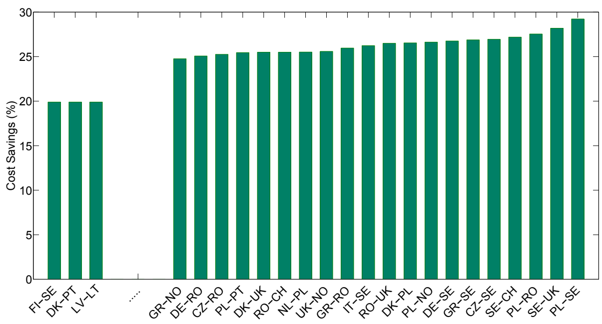

Fig. 6: Cost savings after placing the third HVDC line; first two placed between PT-ES and ES-NO.

C. European Network – Simultaneous Placement of two HVDC lines

In this section, we examine the difference in the obtained results if instead of the sequential approach described in the previous sections, we followed a more parallel approach. In total we ran 122’760 instances of the DC-OPF in order to identify which pair of HVDC lines would result in the maximum cost savings[4]. As shown in Fig. 7, the results coincide with results obtained in Section VI-B. Object of future work will be the derivation of the theoretical guarantees which would show that the sequential approach proposed in this paper is equivalent to the simultaneous placement of an equal number of HVDC lines.

VII. Conclusions and Outlook

Concluding, in this paper we derive a rule for HVDC line placement in order to maximize social welfare. Assuming linear generator costs, we show that for N congested lines, there exist exactly $N+1$ marginal generators. HVDC lines connecting a high-cost with a low-cost marginal generator lead to the maximization of the social welfare. This proposition helps placing an upper bound on the line installation costs, when searching for an optimal HVDC placement, and could also be used in optimization algorithms for bounding the search space. Any solution with installation costs higher than the line connecting marginal generators should be discarded. It also shows that an optimal HVDC placement is not necessarily along the overloaded lines. Instead, for maximizing social welfare, the position of the marginal generators should be taken into account.

Furthermore, an algorithm is outlined which can identify the “optimal” placement between any possible pair of nodes in a single iteration. Additional iterations are necessary in case different line capacities need to be considered, or the line installation costs are taken into account. The proposed approach is also able to predict a lower bound of the expected cost savings after the line placement. The validity of the method is demonstrated with two case studies, one on a 10-bus network and one on a simplified European network.

Future work will seek theoretical guarantees which show that the sequential approach for the placement followed in this paper is equivalent to placing simultaneously an equal number of lines. It will further extend the algorithm so as to consider snapshots where different marginal generators appear.

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[4] Placing 3 HVDC lines simultaneously would result in the investigation of 20’214’480 possible combinations for the 32-node system. This would have led to three orders of magnitude longer computation time.