

# Remove barriers for Machine Learning Applications in Power Systems

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Joint work with Andreas Venzke,  
Georgios Misyris, Jochen Stiasny,  
and with Guannan Qu, Steven Low

Slides available at [www.chatziva.com/spyros\\_c3ai.pdf](http://www.chatziva.com/spyros_c3ai.pdf)

# Machine Learning for Power Systems: Why?

## 1. Extremely fast

- computation within only a **few milliseconds**  
100x – 1000x faster than conventional methods → we can run 1'000 possible scenarios and get a good estimate vs. running only 1 scenario

## 2. Good alternative if we do not have full knowledge of the actual model

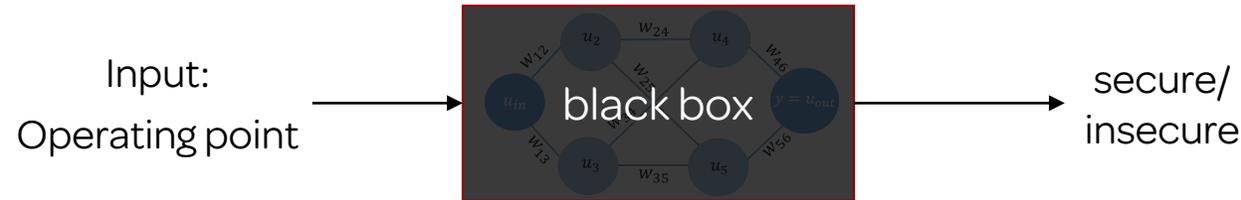
- Handle **very complex systems**
- **Infer** from incomplete data



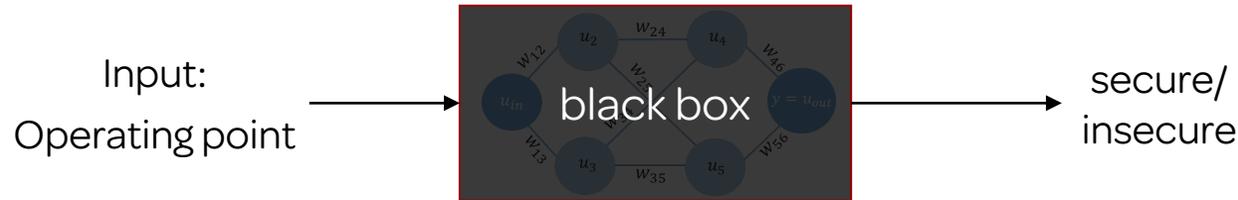
**But: Would an Operator ever trust AI in the Control Room?**

**Our work:** Remove the barriers and build **trustworthy AI-tools**

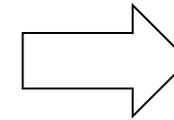
# ML Barriers for Power systems



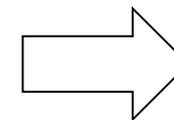
1. Why would we use a “**black box**” to decide about a **safety-critical application**?
2. **Accuracy is a purely statistical** performance metric.  
Who guarantees that the Neural Network can handle well previously unseen operating points?
3. Why would we depend on **discrete and incomplete data**, when we have developed **detailed physical models** over the past 100 years?



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**Neural Network verification:**  
guarantees for the NN performance!



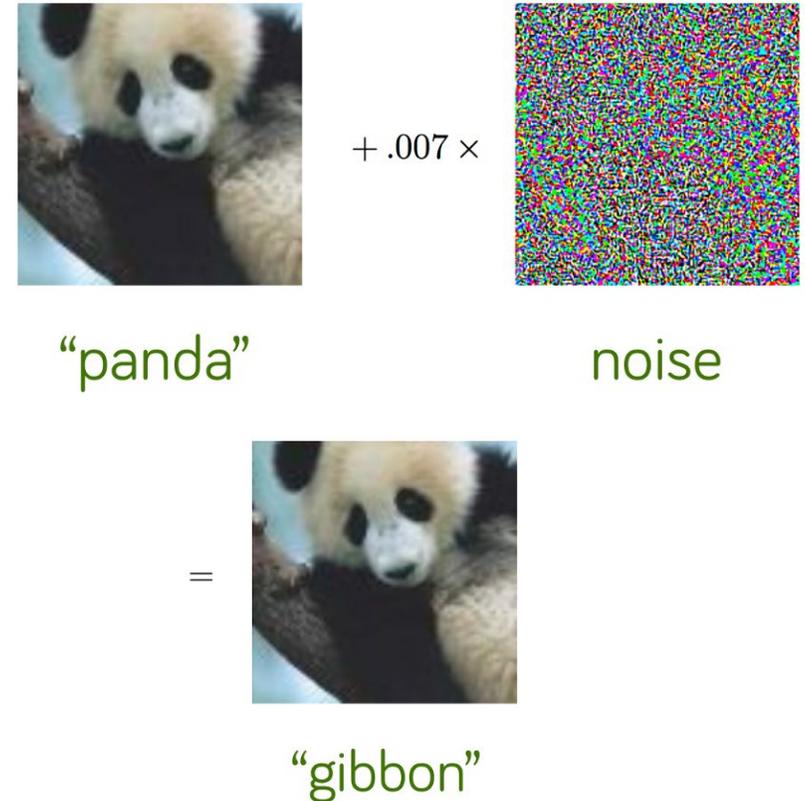
Physics-Informed Neural Networks

# Accuracy is purely statistical!

(or why is Neural Network Verification important?)

**NN accuracy:** Until recently, the only way to assess the performance of neural networks

1. Challenge #1: No way to guarantee what the output is for a **continuous** range of inputs
2. Challenge #2: The **test database** determines the performance of the neural network
  - If the test data come from the same simulations as your training data → accuracy can be deceptively high.  
Would it be equally high for the whole input domain?
3. Challenge #3: No way to systematically identify **adversarial examples**



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“panda”

+ .007 ×



noise

=



“gibbon”



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**Neural network verification:**

1. Can guarantee the output for a **continuous** range of inputs
2. Is **not dependent** on the quality of the test database
3. Can **systematically identify** adversarial examples

This talk:

1. Verification of **Classification Neural Networks** for Power Systems
2. Verification of **Regression Neural Networks**
3. **Physics-Informed** Neural Networks

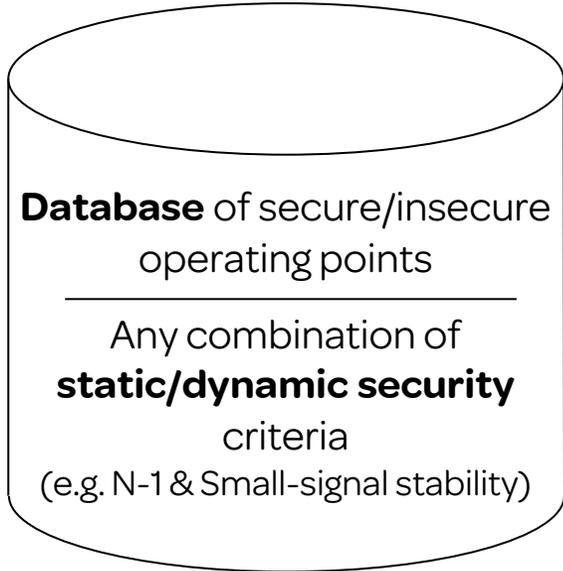
# Neural Network Verification

## for classification NNs in Power Systems

A. Venzke, S. Chatzivasileiadis. Verification of Neural Network Behaviour: Formal Guarantees for Power System Applications. In *IEEE Transactions on Smart Grid*, vol. 12, no. 1, pp. 383-397, Jan. 2021, <https://arxiv.org/pdf/1910.01624.pdf>

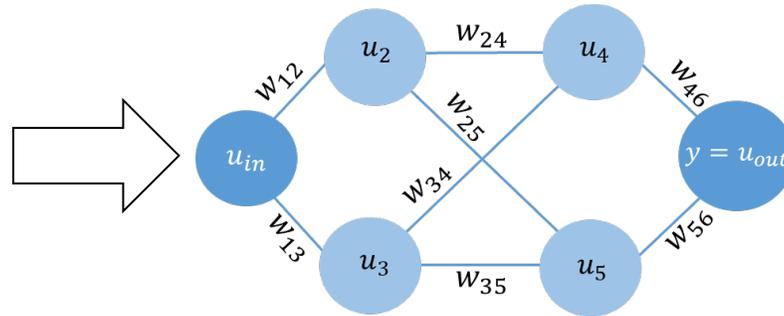
V. Tjeng, K. Y. Xiao, and R. Tedrake, "Evaluating robustness of neural networks with mixed integer programming," in International Conference on Learning Representations (ICLR 2019), 2019

# Guiding Application: Security Assessment with Neural Networks



1. Split the database in a training set and a test set

## Approaches proposed up to now

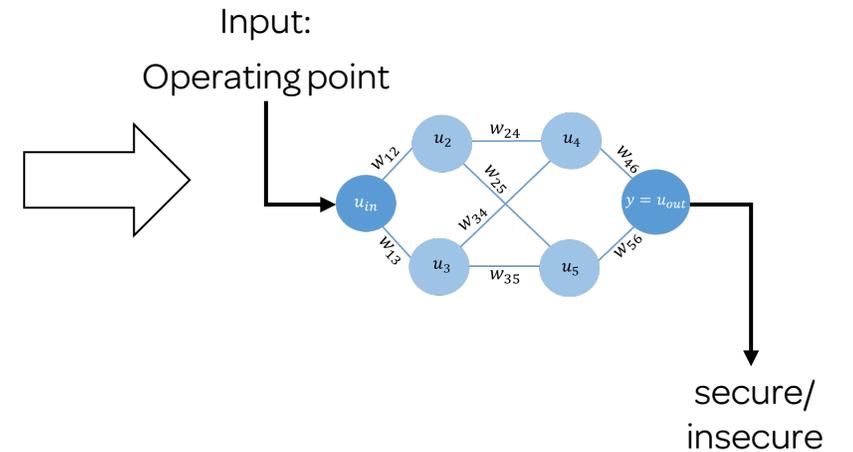


2. Train a neural network

3. Test the neural network

4. Is accuracy high enough?

## 5. Use the NN



### NN Output:

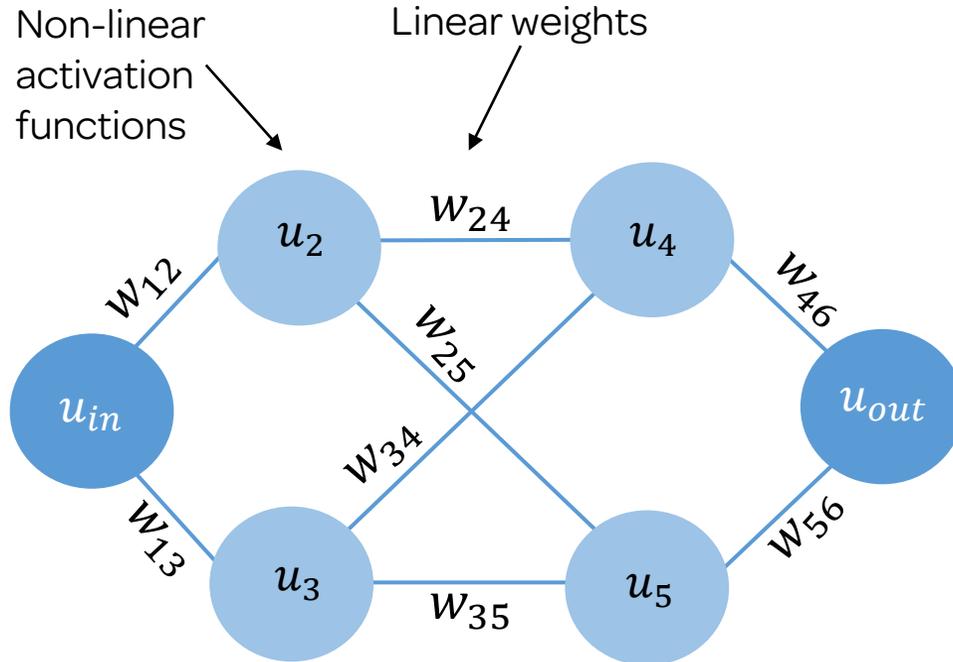
Binary classification:  
**secure/insecure**

**Extremely fast:** up to 100x-1'000x faster

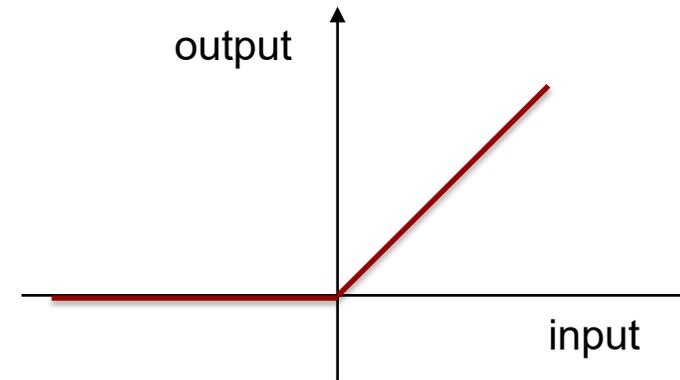
# Neural Network Verification: HOW?

1. **Exact transformation:** Convert the neural network to a **set of linear equations with binaries**
  - The Neural Network can be included in a mixed-integer linear program
2. Formulate an **optimization** problem (MILP) and solve it → certificate for NN behavior
3. Assess if the neural network output complies with the ground truth

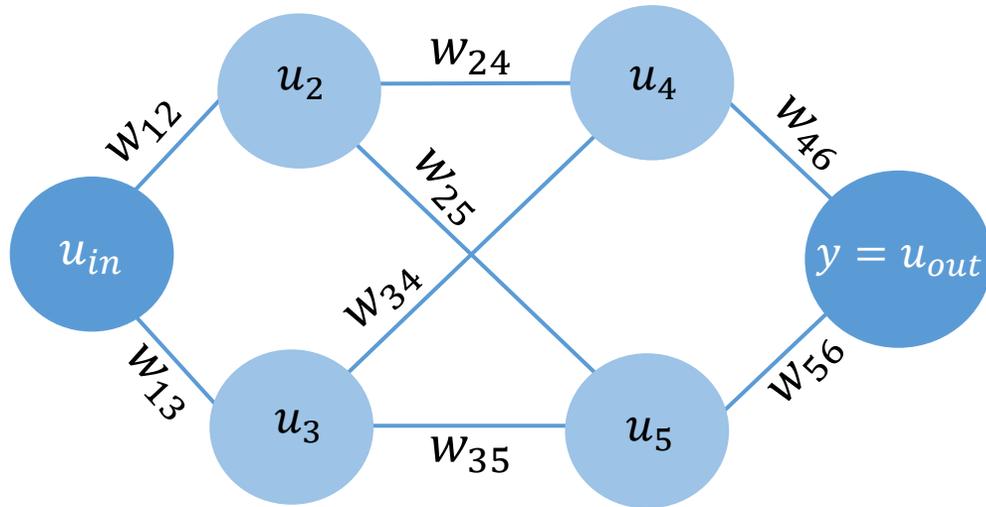
# From Neural Networks to Mixed-Integer Linear Programming



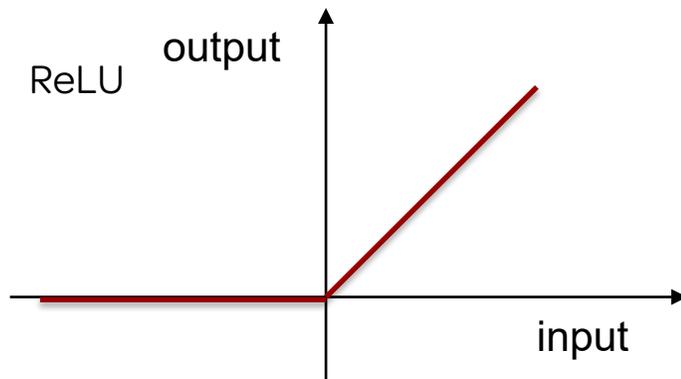
- Most usual activation function: ReLU
- ReLU: Rectifier Linear Unit



# From Neural Networks to Mixed-Integer Linear Programming

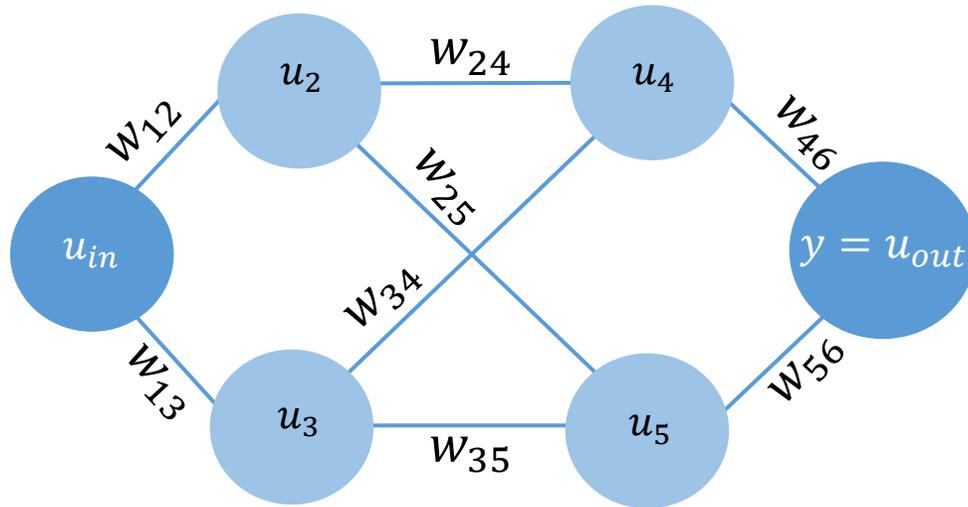


- Linear weights
- On every node: a non-linear activation function
  - ReLU:  $u_j = \max(0, w_{ij}u_i + b_i)$
- But ReLU can be transformed to a piecewise linear function with binaries



MILP

# From Neural Networks to Mixed-Integer Linear Programming



- Input: Active power gen. setpoints

$$\mathbf{x} = [p_{g1}, p_{gi}, \dots, p_{gN}]^T$$

- Output
  - Binary classification: safe/unsafe
  - Output vector  $y$  with two elements:

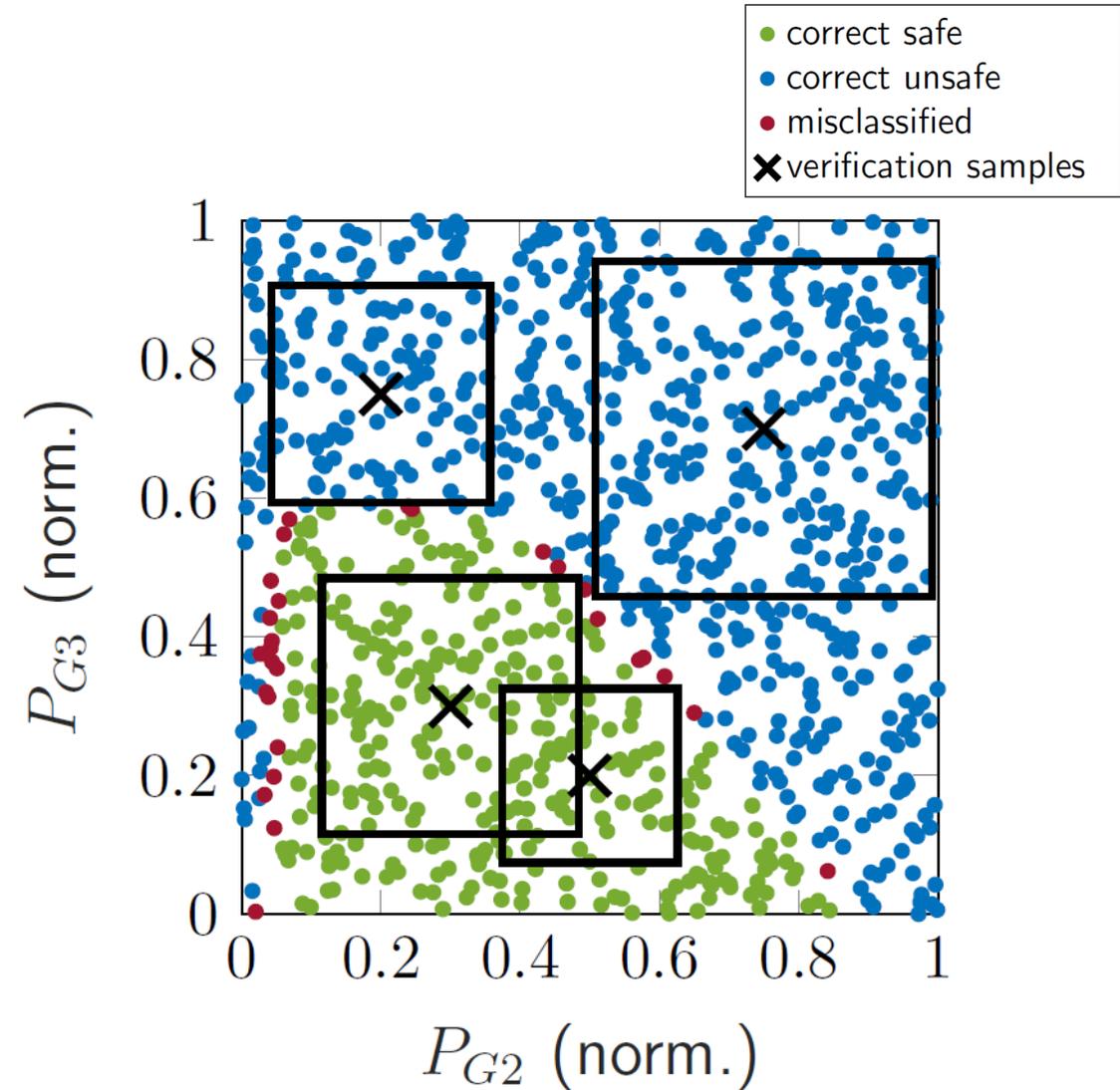
$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad \begin{array}{l} \bullet \text{ } y_1 \geq y_2: \text{safe} \\ \bullet \text{ } y_1 < y_2: \text{unsafe} \end{array}$$

# Certify the output for a continuous range of inputs

- We assume a given input  $x_{\text{ref}}$  with classification  $y: y_1 > y_2$

1. For distance  $\epsilon$  evaluate if input  $x$  exists with different classification  $y_2$

$$\begin{aligned}
 & \max_{x,y} \quad y_2 - y_1 \\
 & \text{s.t.} \quad y = NN(x) \\
 & \quad \quad |x - x_{\text{ref}}|_{\infty} \leq \epsilon
 \end{aligned}$$



# Adversarial examples in safety-critical systems

Original Image



DL Classification: Green Light

Adversarial Example



DL Classification: Red Light

Changing one  
pixel here

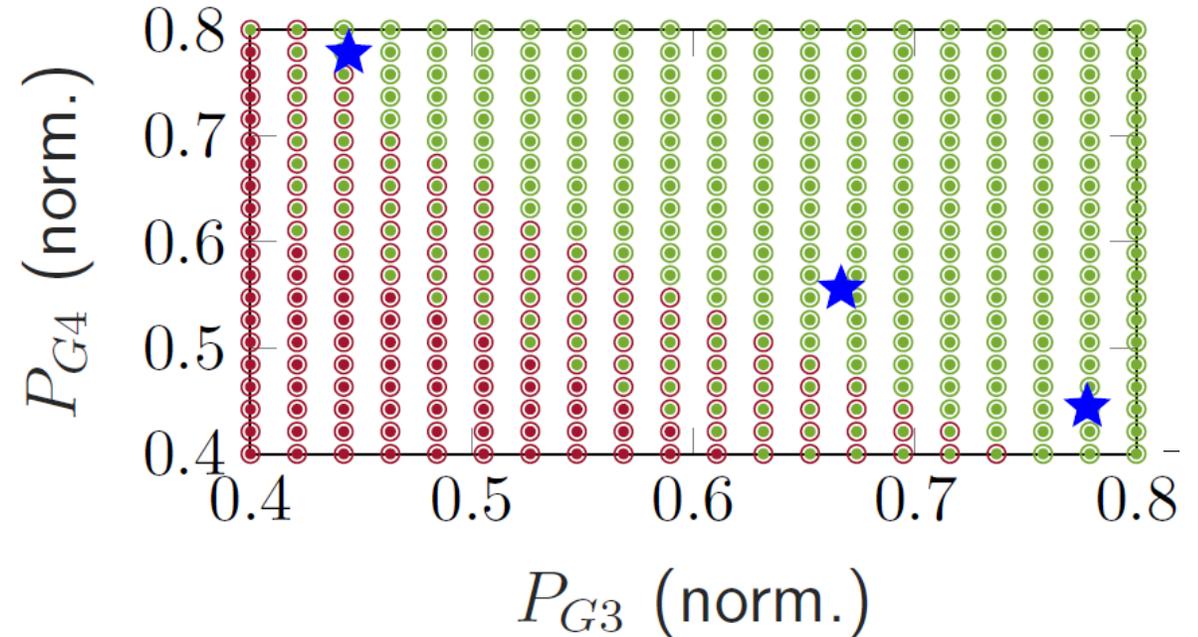
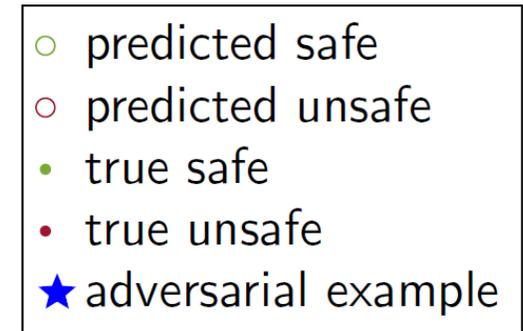
source: Wu et al. A game-based approximate verification of deep neural networks with provable guarantees. arXiv:1807.03571.

- Adversarial examples exist in many (deep) learning applications
- Major barrier for adoption of machine learning techniques in safety-critical systems!

# Systematically identify adversarial examples

- We assume a given input  $\mathbf{x}_{\text{ref}}$  with classification  $y: y_1 > y_2$
2. Minimize distance  $\epsilon$  from  $\mathbf{x}_{\text{ref}}$  to input  $\mathbf{x}$  with classification  $y_2$

$$\begin{aligned}
 \min_{\mathbf{x}, y, \epsilon} \quad & \epsilon \\
 \text{s.t.} \quad & y = NN(\mathbf{x}) \\
 & |\mathbf{x} - \mathbf{x}_{\text{ref}}|_{\infty} \leq \epsilon \\
 & y_2 \geq y_1
 \end{aligned}$$



# Learning OPF: **Worst-case Guarantees for Regression Neural Networks**

A. Venzke, G. Qu, S. Low, S. Chatzivasileiadis, Learning Optimal Power Flow: Worst-case Guarantees for Neural Networks. **Best Student Paper Award** at IEEE SmartGridComm 2020. <https://arxiv.org/pdf/2006.11029.pdf>

# Quick Reminder: DC Optimal Power Flow

- **Objective:** find the minimum cost generation dispatch
- **Input:** Varying load demand at different nodes
- Considered constant: generator costs; system topology

$$\min_{\mathbf{p}_g, \boldsymbol{\theta}} \quad \mathbf{c}^T \mathbf{p}_g$$

Minimizes generation cost

$$\text{s.t.} \quad \mathbf{M}_g \mathbf{p}_g - \mathbf{M}_d \mathbf{p}_d = \mathbf{B}_{\text{bus}} \boldsymbol{\theta}$$

Nodal power balance

$$-\mathbf{p}_{\text{line}}^{\max} \leq \mathbf{B}_{\text{line}} \boldsymbol{\theta} \leq \mathbf{p}_{\text{line}}^{\max}$$

Transmission line limits

$$\mathbf{p}_g^{\min} \leq \mathbf{p}_g \leq \mathbf{p}_g^{\max}$$

Generator limits

# Quick Reminder: DC Optimal Power Flow

- **Objective:** find the minimum cost generation dispatch
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- Considered constant: generator costs; system topology

Several recent approaches in the literature that apply Neural Networks for solving the DC-OPF

- Demonstrate up to **100x speedup**
- But **no performance guarantees**

$$\min_{\mathbf{p}_g, \boldsymbol{\theta}} \mathbf{c}^T \mathbf{p}_g$$

Minimizes generation cost

$$\text{s.t. } \mathbf{M}_g \mathbf{p}_g - \mathbf{M}_d \mathbf{p}_d = \mathbf{B}_{\text{bus}} \boldsymbol{\theta}$$

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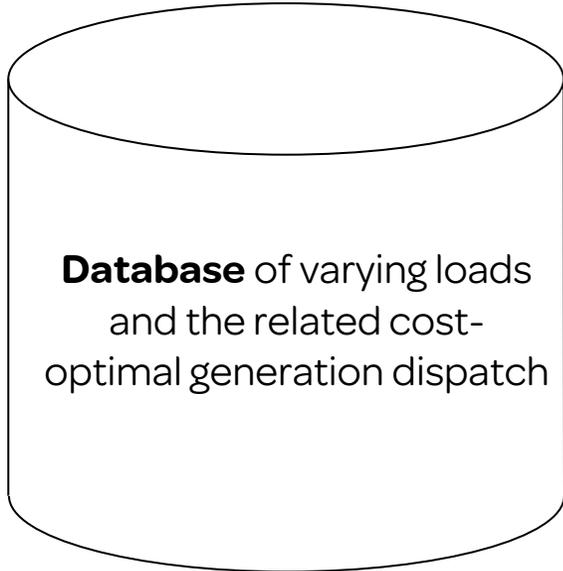
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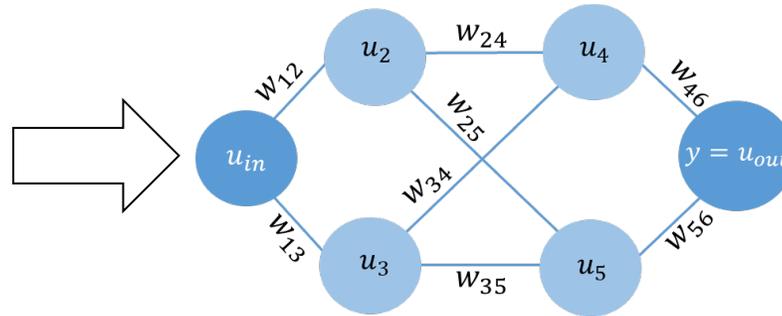
Generator limits

# Guiding Application for Regression NN: Learning OPF



1. Split the database in a training set and a test set

## Approaches proposed up to now

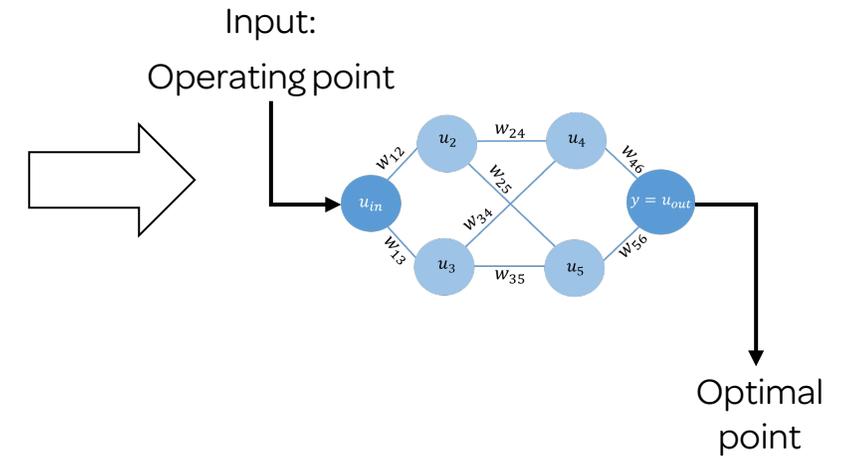


2. Train a neural network

3. Test the neural network

4. Is accuracy high enough?

## 5. Use the NN



### NN Output:

Optimal generation dispatch

### Extremely fast:

up to 100x faster

# Part I: Maximum limit-violations

1. Maximum violation of generator limits

$$\nu_g = \max(\hat{\mathbf{p}}_g - \mathbf{p}_g^{\max}, \mathbf{p}_g^{\min} - \hat{\mathbf{p}}_g, \mathbf{0})$$

$$\begin{aligned} \max \quad & \nu_g \\ \text{s.t.} \quad & \mathbf{A}_d \mathbf{p}_d \leq \mathbf{b}_d \quad \text{Convex polytope as input domain } \mathcal{D} \\ & \hat{\mathbf{p}}_g = NN(\mathbf{p}_d) \quad \text{Mixed-integer reformulation of trained NN} \end{aligned}$$

Example:

$$0.6 \mathbf{p}_d^{\max} \leq \mathbf{p}_d \leq 1.0 \mathbf{p}_d^{\max}$$

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2. Maximum violation of line limits

$$\nu_{\text{line}} = \max(\underbrace{|\mathbf{B}_{\text{line}} \tilde{\mathbf{B}}_{\text{bus}}^{-1} (\mathbf{M}_g \hat{\mathbf{p}}_g - \mathbf{M}_d \mathbf{p}_d)^{\text{nsb}}|}_{\text{Line flow equations for DC-OPF based on PTDFs}} - \mathbf{p}_{\text{line}}^{\max}, \mathbf{0})$$

Line flow equations for DC-OPF based on PTDFs

$$\begin{aligned} \max \quad & \nu_{\text{line}} \\ \text{s.t.} \quad & \mathbf{A}_d \mathbf{p}_d \leq \mathbf{b}_d \quad \text{Convex polytope as input domain } \mathcal{D} \\ & \hat{\mathbf{p}}_g = NN(\mathbf{p}_d) \quad \text{Mixed-integer reformulation of trained NN} \end{aligned}$$

Worst violation over the **whole training dataset**  
(training+test set)

Our algorithm: **provable**  
worst-case guarantee over  
the **whole input domain**

	Empirical lower bound		Exact worst-case guarantee	
Test cases	$\nu_g$ (MW)	$\nu_{line}$ (MW)	$\nu_g$ (MW)	$\nu_{line}$ (MW)
<i>case9</i>				
<i>case30</i>				
<i>case39</i>				
<i>case57</i>				
<i>case118</i>				
<i>case162</i>				
<i>case300</i>				

$\nu_g$  Maximum violation of generator limits

$\nu_{line}$  Maximum violation of line limits

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<i>case9</i>	2.5	1.8	2.8	1.9
<i>case30</i>	1.7	0.6	3.6	3.1
<i>case39</i>	51.9	37.2	270.6	120.0
<i>case57</i>	4.2	0.0	23.7	0.0
<i>case118</i>	149.4	15.6	997.8	510.8
<i>case162</i>	228.0	180.0	1563.3	974.1
<i>case300</i>	474.5	692.7	3658.5	3449.3

$\nu_g$  Maximum violation of generator limits

$\nu_{line}$  Maximum violation of line limits

Over the whole input domain **violations can be much larger** (here ~7x) compared to what has been estimated empirically on the dataset



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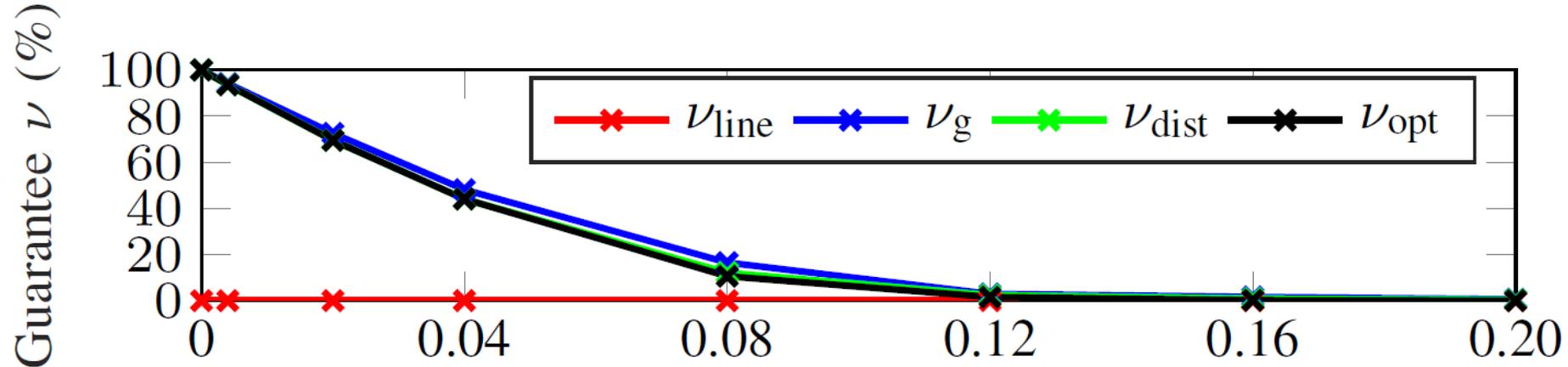
Our method provides **guarantees that no NN output will violate the line limits** over the whole input domain

## How can we reduce the worst-case violations?

- From our experiments with DC-OPF in 7 different test power systems, we observed that the **worst-case violations occur at the boundary of the input domain**
- Possible solution:
  1. Train on a larger input domain
  2. Use the NN on a subdomain of the original training input

# Reducing the worst-case violations

100% on y-axis =  
Worst-case violation  
for  $\delta = 0$



(b) *case57*: Input domain reduction  $\delta$  (-)

- Input domain used for training

$$0.6 \mathbf{p}_d^{\max} \leq \mathbf{p}_d \leq 1.0 \mathbf{p}_d^{\max}$$

- Input domain for using the NN (and where worst-case violations were evaluated)

$$(0.6 + \delta) \mathbf{p}_d^{\max} \leq \mathbf{p}_d \leq (1.0 - \delta) \mathbf{p}_d^{\max}$$

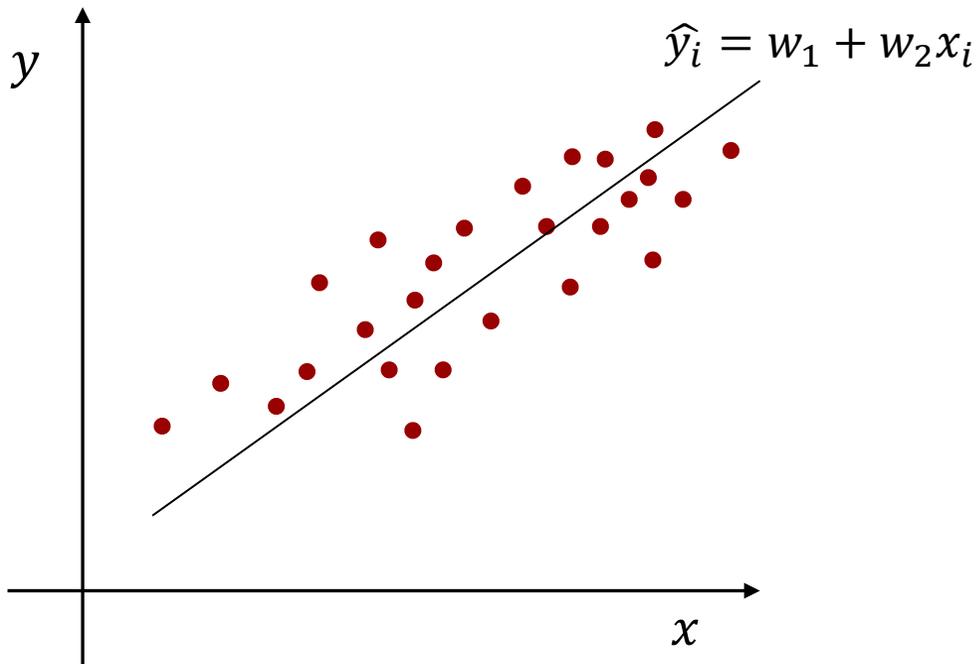
Example: If  $\delta = 0.1$  then  $0.7 \mathbf{p}_d^{\max} \leq \mathbf{p}_d \leq 0.9 \mathbf{p}_d^{\max}$

# Physics-Informed Neural Networks for Power Systems

# Neural Networks: An advanced form of non-linear regression

$y_i$ : actual/correct value

$\hat{y}_i$ : estimated value



**Loss function: Estimate best  $w_1, w_2$  to fit the training data**

$$\begin{aligned} & \min_{w_1, w_2} \|y_i - \hat{y}_i\| \\ \text{s.t.} & \hat{y}_i = w_1 + w_2 x_i \quad \forall i \end{aligned}$$

**Traditional training of neural networks required no information about the underlying physical model. Just data!**

# Physics Informed Neural Networks

- Automatic differentiation: derivatives of the neural network output with respect to the input can be computed during the training procedure
- A differential-algebraic model of a physical system can be included in the neural network training\*
- Neural networks can now exploit knowledge of the actual physical system
- Machine learning platforms such as Tensorflow enable these capabilities

\*M. Raissi, P. Perdikaris, and G. Karniadakis, "Physics-Informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations", *Journal of Computational Physics*, vol.378, pp. 686-707, 2019

# Physics-Informed Neural Networks for Power Systems

“Original”  
Loss function

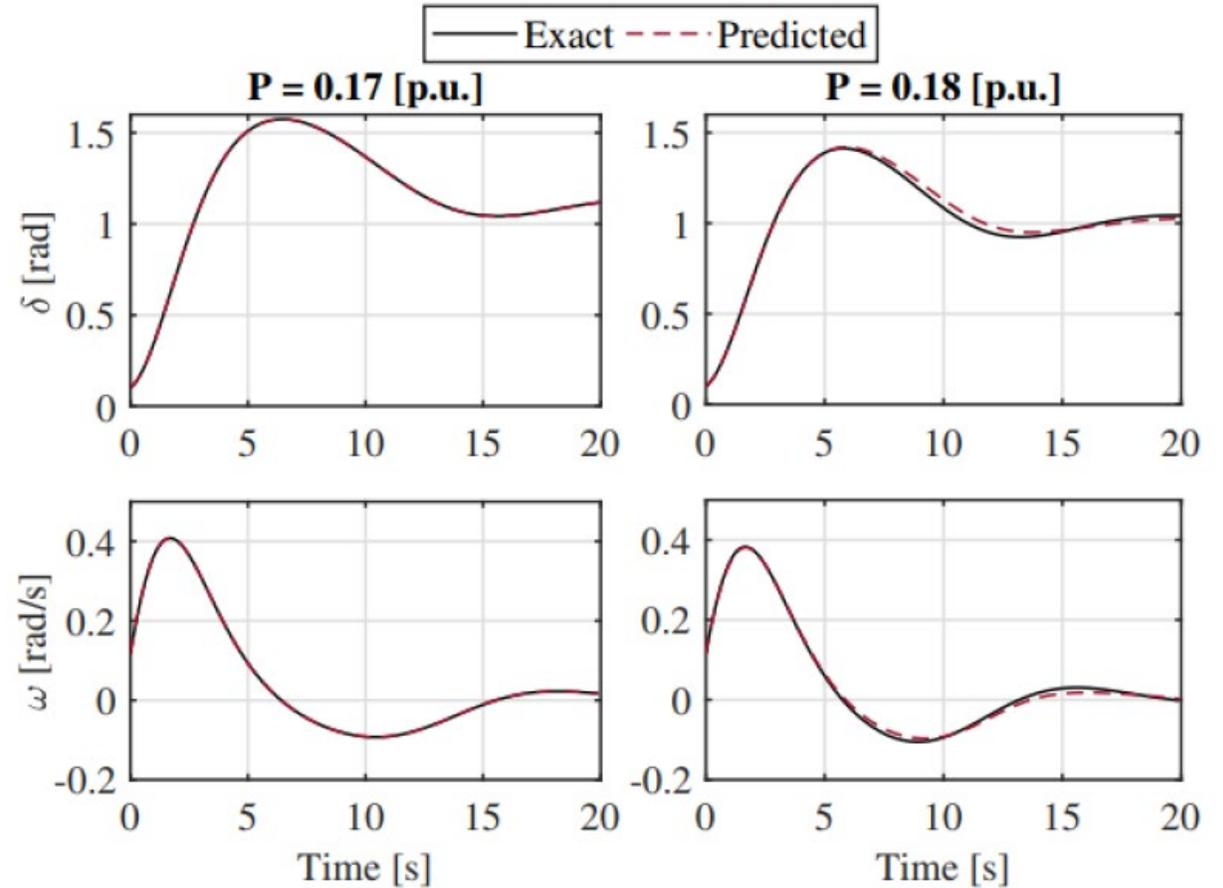
$$\min_{\mathbf{W}, \mathbf{b}} \frac{1}{|N_\delta|} \sum_{i \in N_\delta} |\hat{\delta} - \delta^i|^2 + \frac{1}{|N_f|} \sum_{i \in N_f} |f(\hat{\delta})|^2 \quad (6a)$$

$$s.t. \quad \hat{\delta} = NN(t, P_m, \mathbf{W}, \mathbf{b}) \quad (6b)$$

$$\dot{\hat{\delta}} = \frac{\partial \hat{\delta}}{\partial t}, \quad \ddot{\hat{\delta}} = \frac{\partial^2 \hat{\delta}}{\partial t^2} \quad (6c)$$

$$f(\hat{\delta}) = M \ddot{\hat{\delta}} + D \dot{\hat{\delta}} + A \sin \hat{\delta} - P_m \quad (6d)$$

Swing equation



G. S. Misyris, A. Venzke, S. Chatzivasileiadis, Physics-Informed Neural Networks for Power Systems. Presented at the Best Paper Session of IEEE PES GM 2020. <https://arxiv.org/pdf/1911.03737.pdf>

# Physics-Informed Neural Networks for Power Systems

“Original”  
Loss function

“Physics-Informed”  
term

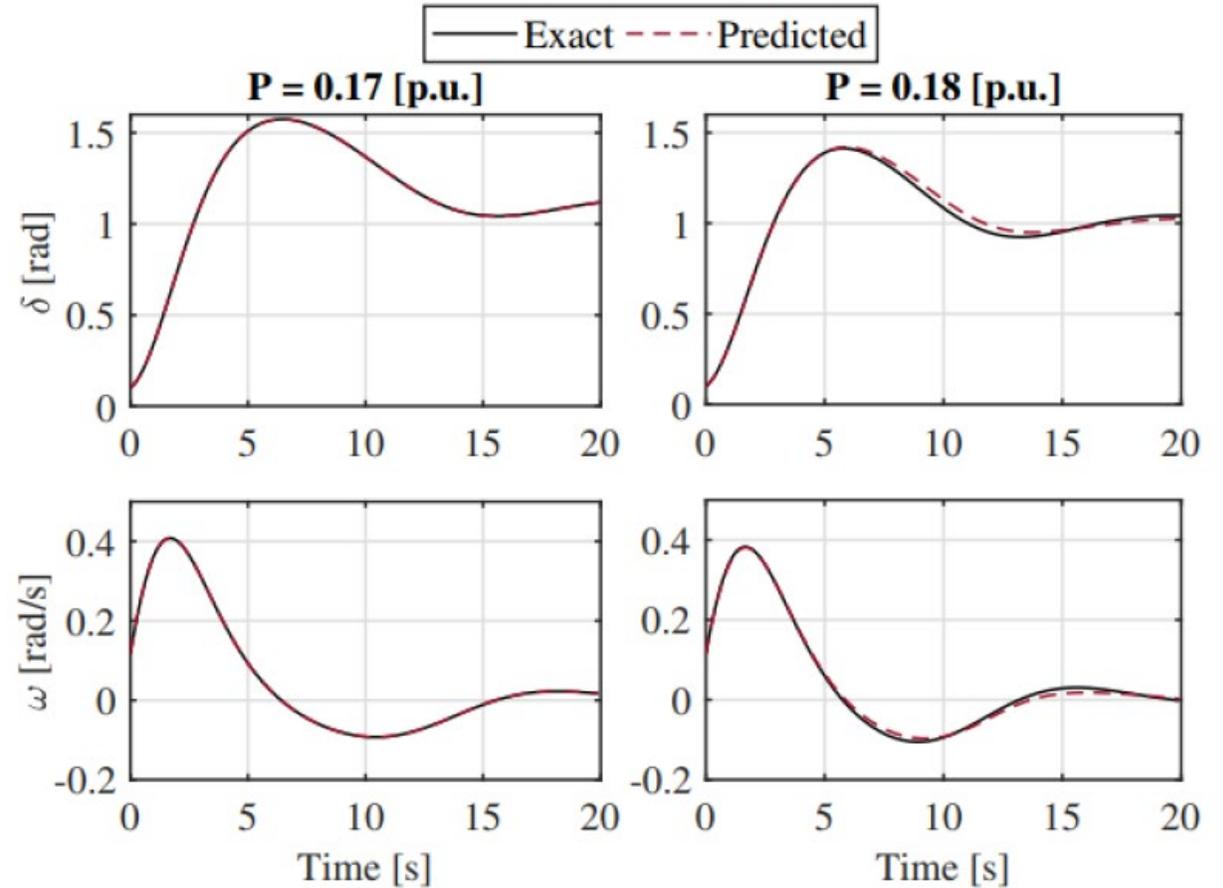
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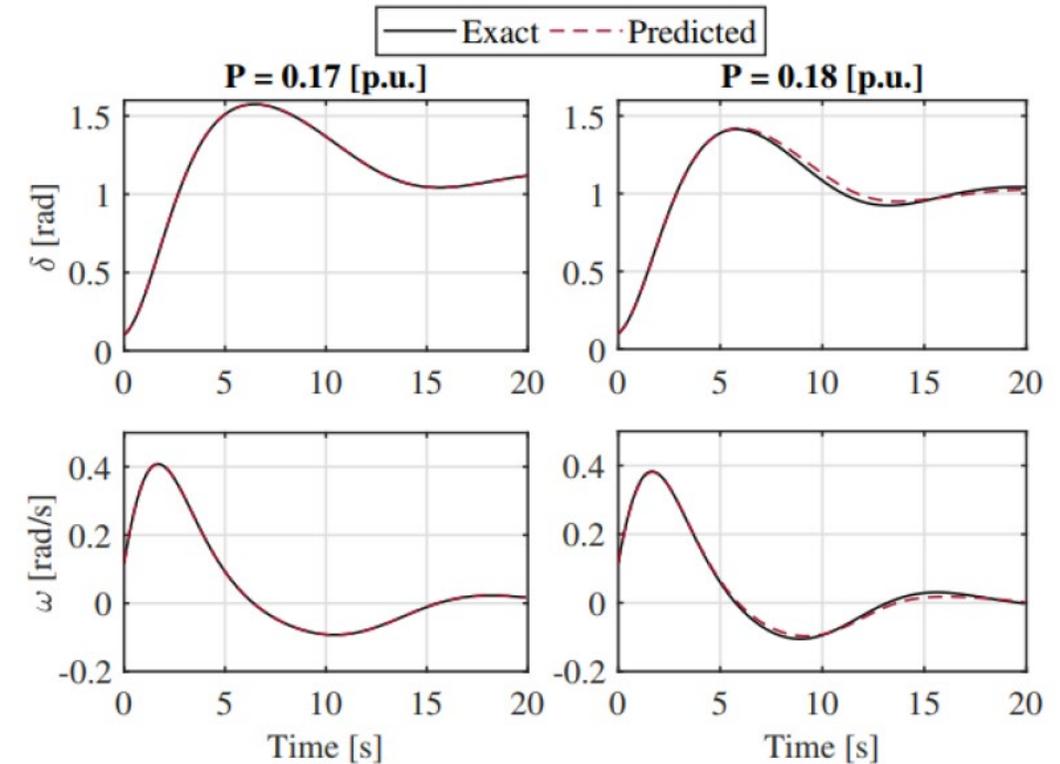
Swing equation



G. S. Misyris, A. Venzke, S. Chatzivasileiadis, Physics-Informed Neural Networks for Power Systems. Presented at the Best Paper Session of IEEE PES GM 2020. <https://arxiv.org/pdf/1911.03737.pdf>

# Physics-Informed Neural Networks for Power Systems

- Physics-Informed Neural Networks (PINN) **can potentially replace** solvers for systems of differential-algebraic equations
- In our example: PINN 87 times faster than ODE solver
- Can **directly estimate** the rotor angle at **any** time instant



Code is available on GitHub: <https://github.com/gmisy/Physics-Informed-Neural-Networks-for-Power-Systems/>

G. S. Misyris, A. Venzke, S. Chatzivasileiadis, Physics-Informed Neural Networks for Power Systems. Presented at the Best Paper Session of IEEE PES GM 2020. <https://arxiv.org/pdf/1911.03737.pdf>

# Wrap-up

- Neural network verification can remove barriers for neural network applications in power systems
- Physics Informed Neural Networks can take advantage of the rich information about existing power system models
- This talk:
  1. Verification of Classification Neural Networks for Power Systems
  2. Provable worst-case guarantees for Regression Neural Networks
    - Application in OPF (and probably any linear program)
  3. Physics-Informed Neural Networks for Power Systems

# Ongoing work

## Exploring a wide range of research directions

1. Contracting Neural-Newton Solver
  - Derive convergence guarantees for Neural Networks that can replace conventional Newton solvers  
[ [www.chatziva.com/publications/Chevalier\\_etal\\_CoNNS\\_ArXiv.pdf](http://www.chatziva.com/publications/Chevalier_etal_CoNNS_ArXiv.pdf) ]
2. Physics-Informed Neural Networks for Fast Dynamic Security Assessment  
[soon on ArXiv, and more user-friendly code on PINNs on github!]
3. Physics-Informed Neural Networks for OPF with worst-case guarantees [soon on ArXiv]
4. Using neural networks to capture previously intractable optimization constraints  
[ <https://arxiv.org/pdf/2103.17004.pdf> ]
5. Accelerating MILPs: using Decision Trees to estimate the active set and drastically reduce the number of binary variable [ <https://arxiv.org/pdf/2010.06344.pdf> ]

and others...

# Open Research Challenges

- **Tractability** for large neural networks
  - Up to now, we have verified NNs with 4 layers and 100 nodes at each layer (NN used for the 162-bus system)
  - We require weight sparsification, bound tightening, and ReLU pruning (remove binary variables) to maintain tractability
- **Retraining** is necessary to avoid adversarial examples
  - The **quality of the training database is crucial** for good performance!
- **Connect verification with ground truth assessment**
- **PINNs**: tractability for larger power systems

# Thank you!



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J. Stiasny, G. S. Misyris, S. Chatzivasileiadis, Physics-Informed Neural Networks for Non-linear System Identification applied to Power System Dynamics. IEEE Powertech 2021.

<https://arxiv.org/pdf/2004.04026.pdf>

Some code available at:

[www.chatziva.com/downloads.html](http://www.chatziva.com/downloads.html)

# Current Work and Open Challenges

- PINNs for System Identification
- PINNs for quick simulation/estimation of converter dynamics
  - Can provide a quick estimate to a multi time-scale or co-simulation platform
- Tractability is an issue
  - Currently capturing all system dynamics for up to an 11-bus system with a single PINN
  - PINNs can also be applied for larger systems, but the computing effort (e.g. for training) will increase as well
  - Need to develop good approaches to maintain a lower computing effort while we scale up to bigger systems