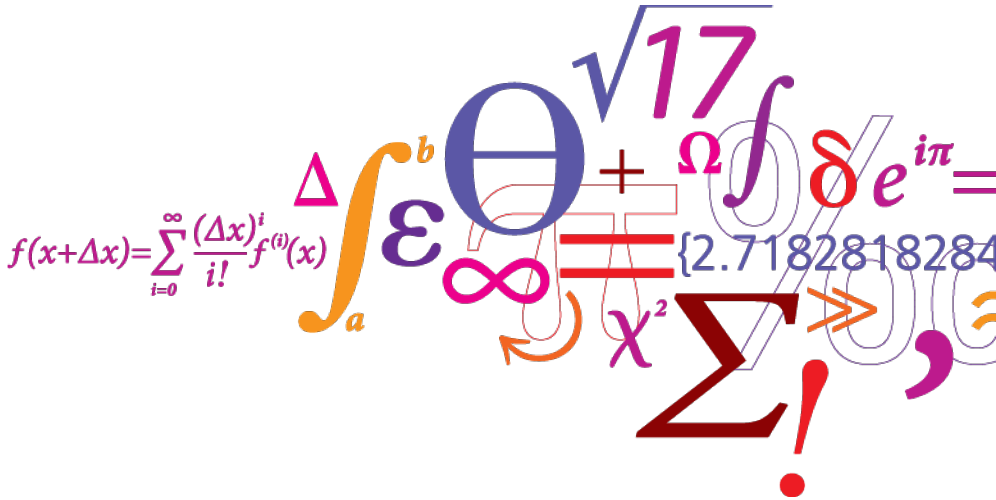
31730: Electric Power Engineering, fundamentals

Transformers, and per unit system

Spyros Chatzivasileiadis

15 September 2017



DTU Electrical Engineering
Department of Electrical Engineering

If not otherwise indicated, all figures are taken from the course textbook: J. D. Glover, T. J. Overbye, M. S. Sarma, Power System Analysis and Design, Sixth Edition - SI, Cengage Learning, 2016

Reviewing previous lecture

- For 5 minutes discuss with the person sitting next to you about:
  - Three main points we discussed in the last lecture
  - One topic or concept that is not so clear and you would like to hear again about it

The goals for today

- Transformer: Basic principles
- Transformer: Main relationships
- Transformer: Equivalent circuit
- Per unit system
- Calculations with per unit

Source: http://www.meppi.com/

Transformers: One of the main reasons that AC power prevailed over DC power

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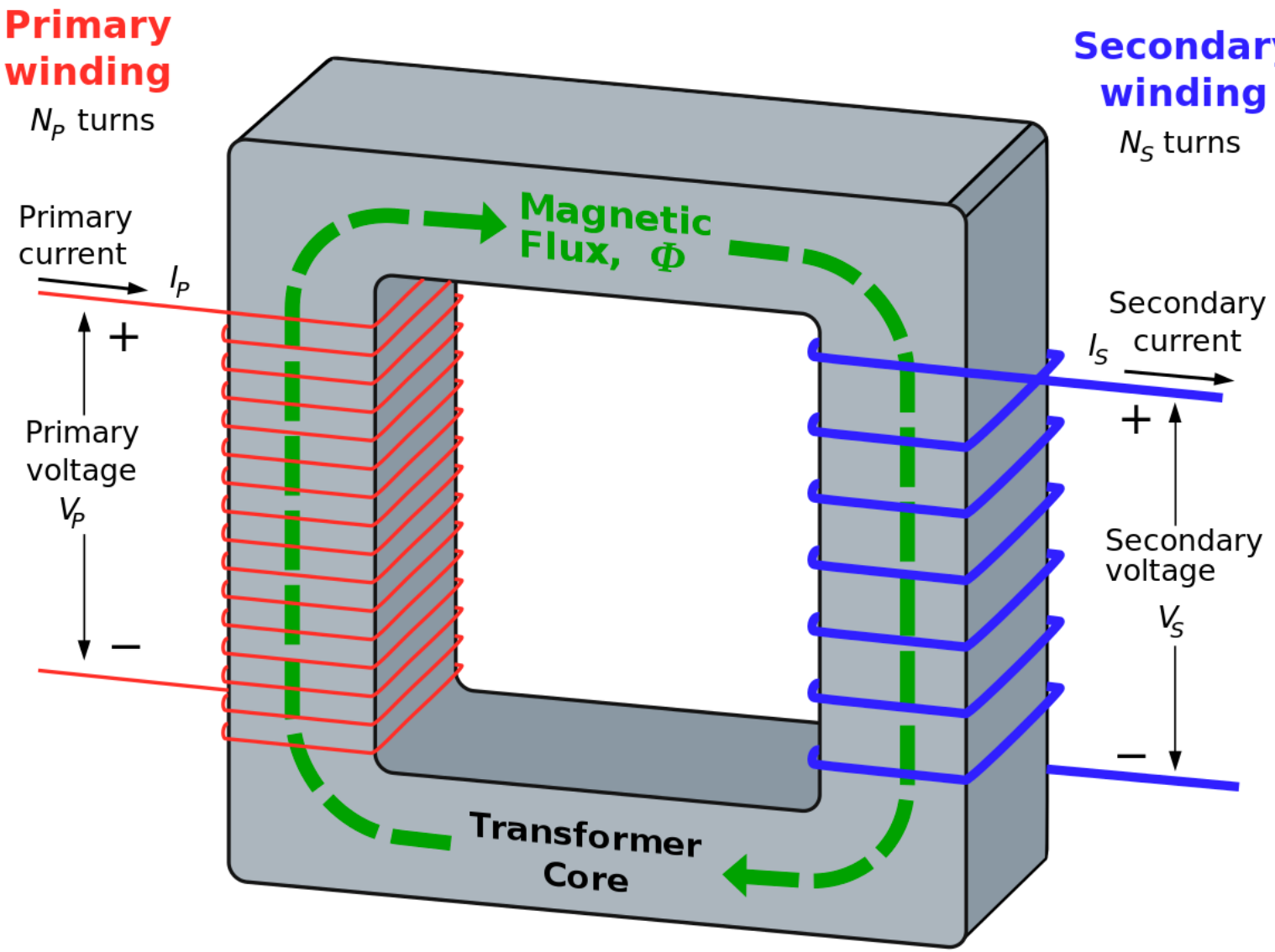
Transformers: One of the main reasons that AC power prevailed over DC power

Why?



Source: https://commons.wikimedia.org/w/index.php?curid=27407689
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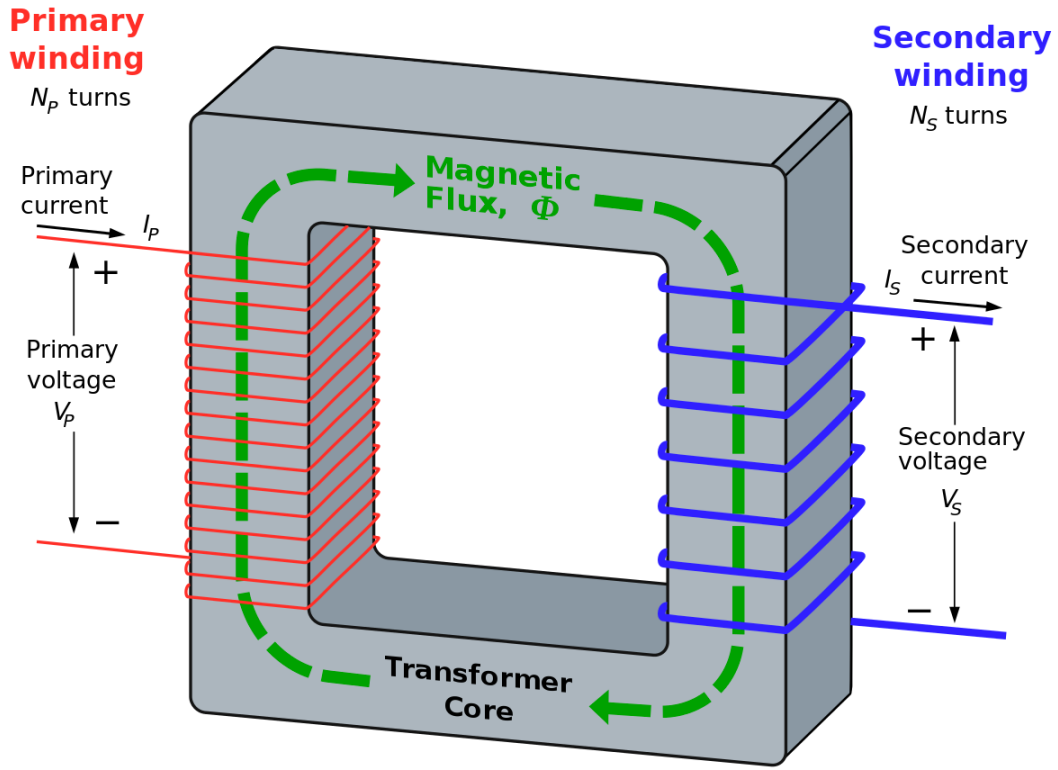
Transformer: (very) basic principles



- Two windings
- One magnetic core

- Ampere's law: electric current induces magnetic field
  - right-hand rule

$$\oint H dl = I_{\text{enc}}$$

- Faraday's law (and Lenz's law): *time-varying* magnetic field induces an electromotive force (voltage) opposing the original magnetic flux

$$e(t) = -N\frac{d\phi(t)}{dt}$$

Source: https://commons.wikimedia.org/w/index.php?curid=27407689
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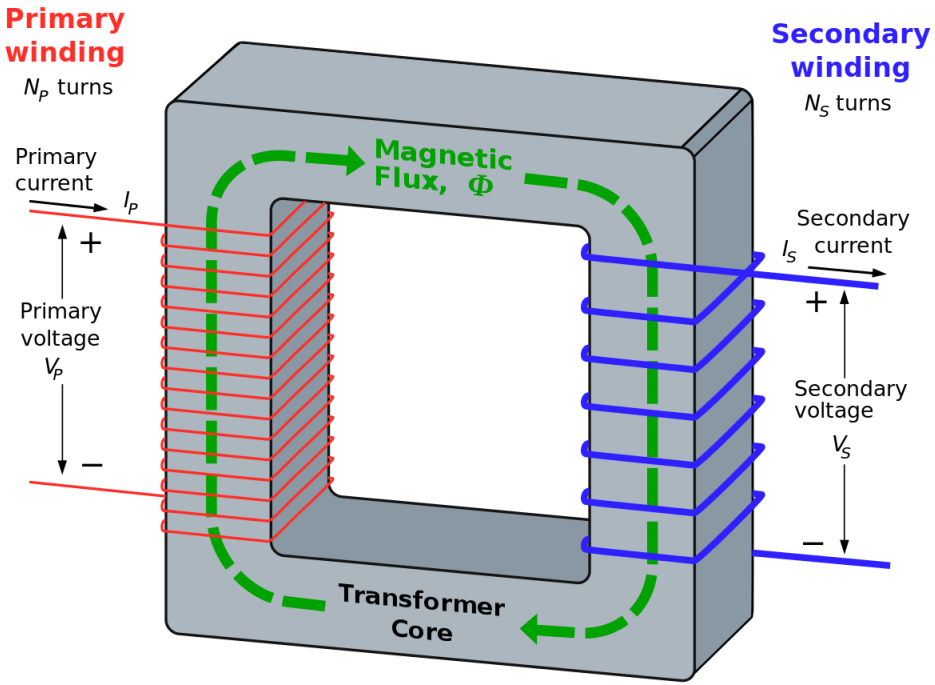
Ideal transformer: Main relationships



- Voltage proportional to the turns-ratio

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \alpha_t$$

- Current inversely proportional to the turns-ratio

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{\alpha_t}$$

- Complex power is constant (conservation of power in an ideal transformer)

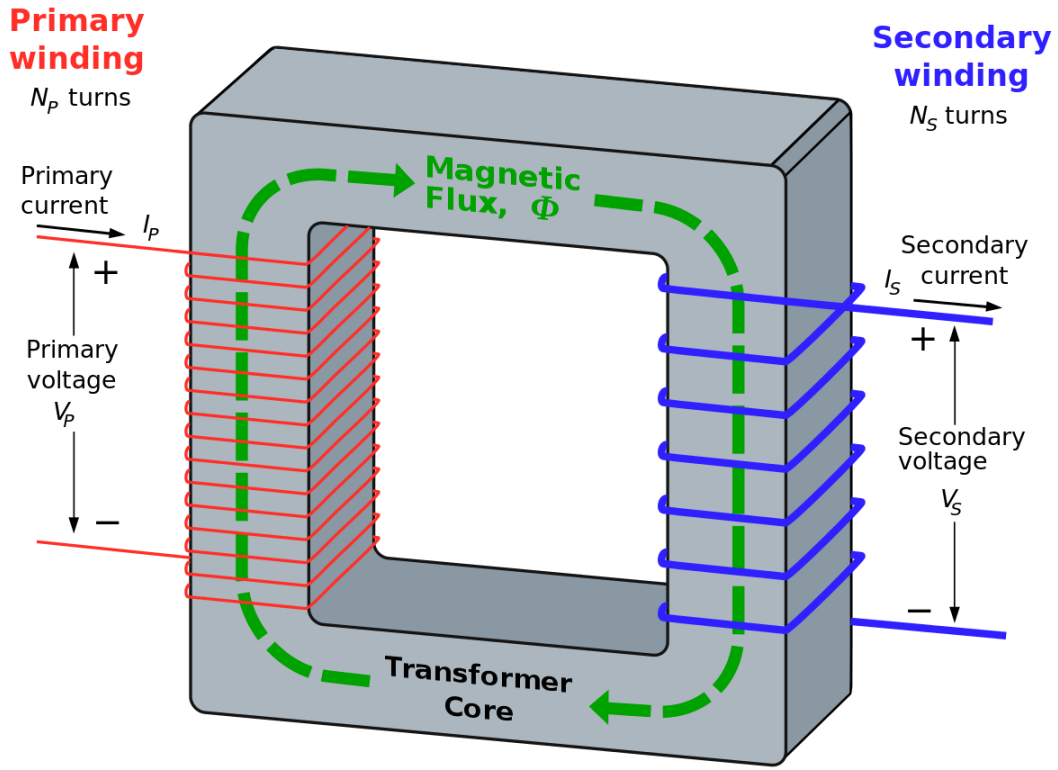
$$S_1 = S_2$$

- Impedance proportional to α_t^2

$$\frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2}\right)^2 = \alpha_t^2$$

Source: https://commons.wikimedia.org/w/index.php?curid=27407689

Use of Transformers (not exhaustive list)



- Widely used to transform towards higher or lower voltages
- Special transformers:
  - Tap-changing transformers: variable turns-ratio → vary the voltage magnitude in the secondary
  - Phase-shifting transformers: change the voltage phase angle
- Galvanic isolation: the two circuits are not electrically connected
  - 1:1 transformers are used to isolate certain "critical" circuits for increased protection

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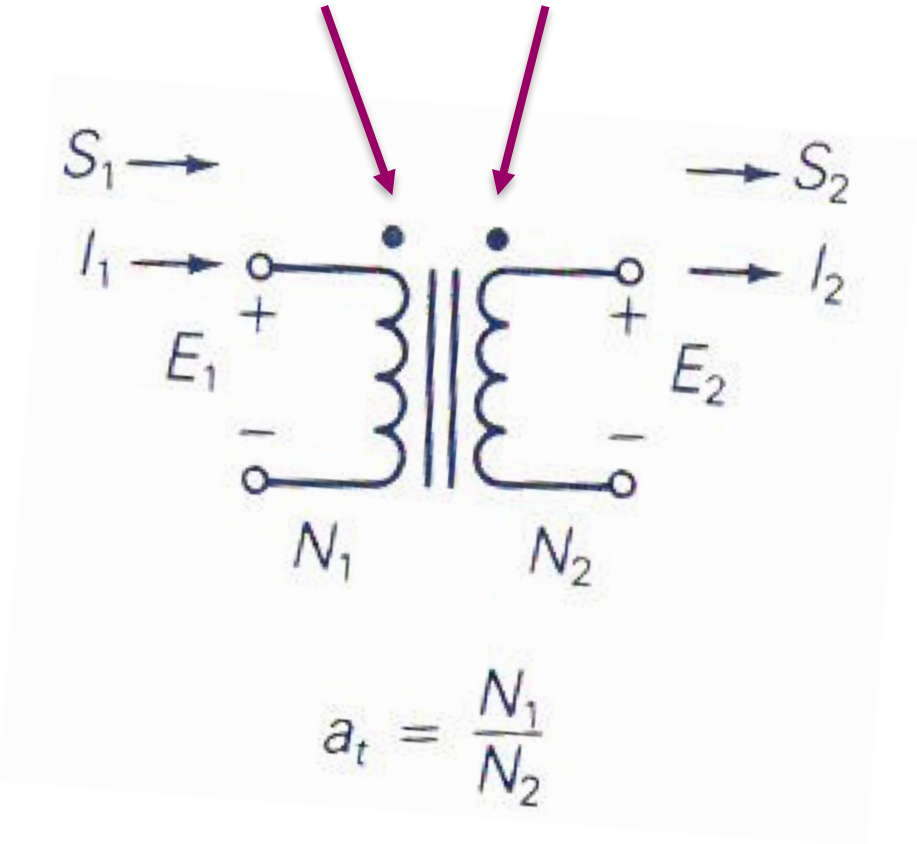
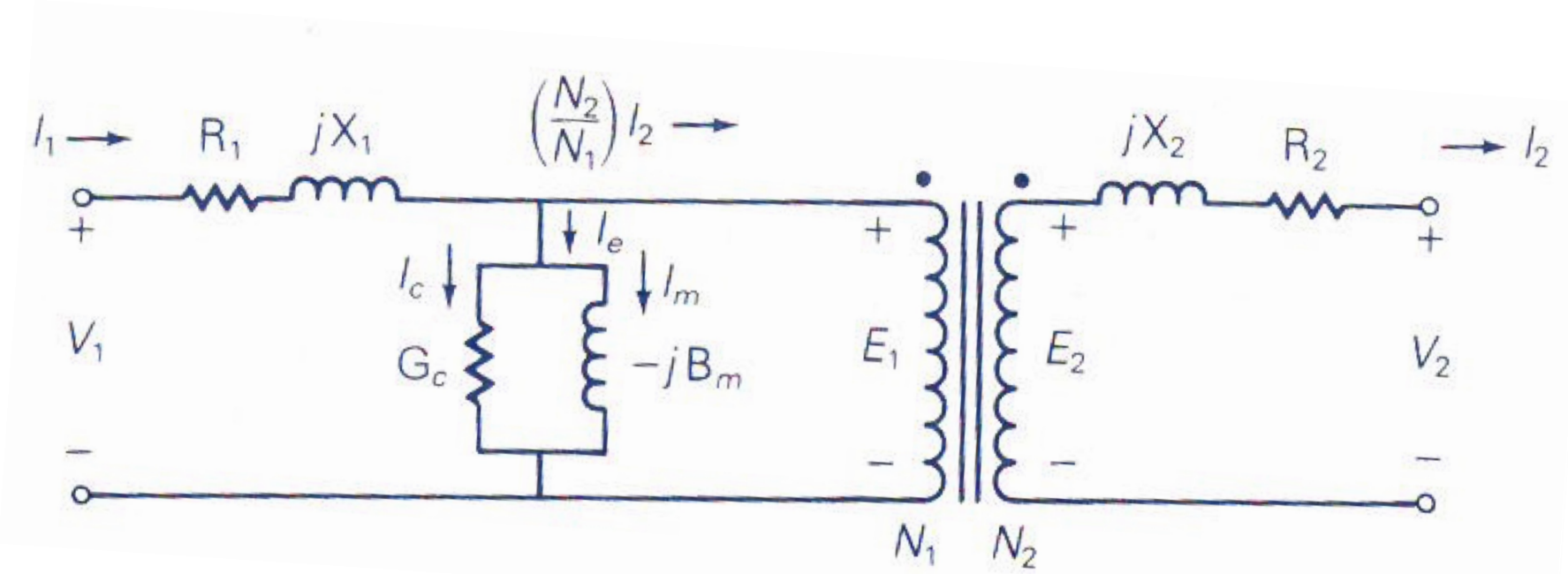
Equivalent circuit of an ideal single-phase transformer



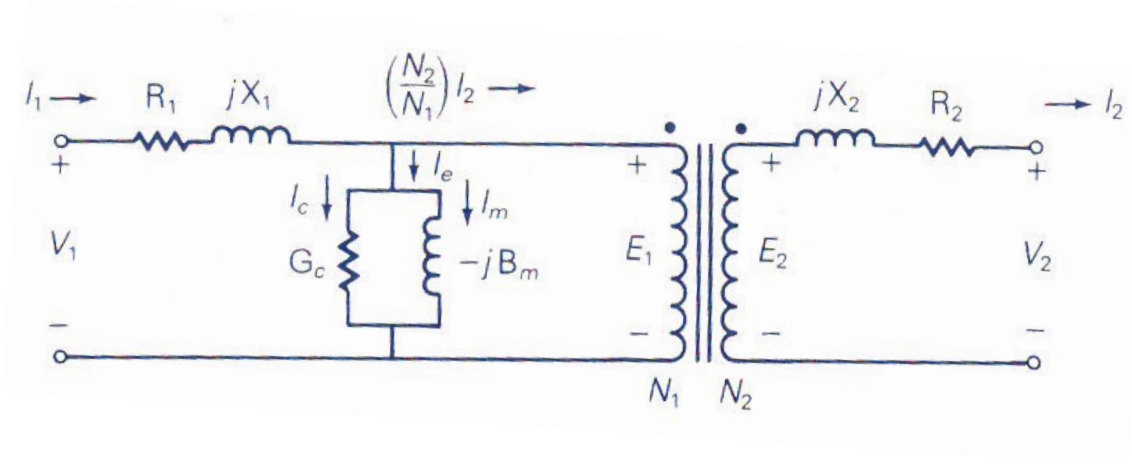
- Dots show the polarity of the voltage
- E_1 and E_2 have their + polarities at the dotted terminals

Equivalent circuit of a single-phase transformer



A practical transformer operating in sinusoidal steady state is equivalent to an ideal transformer with external impedance and admittance branches

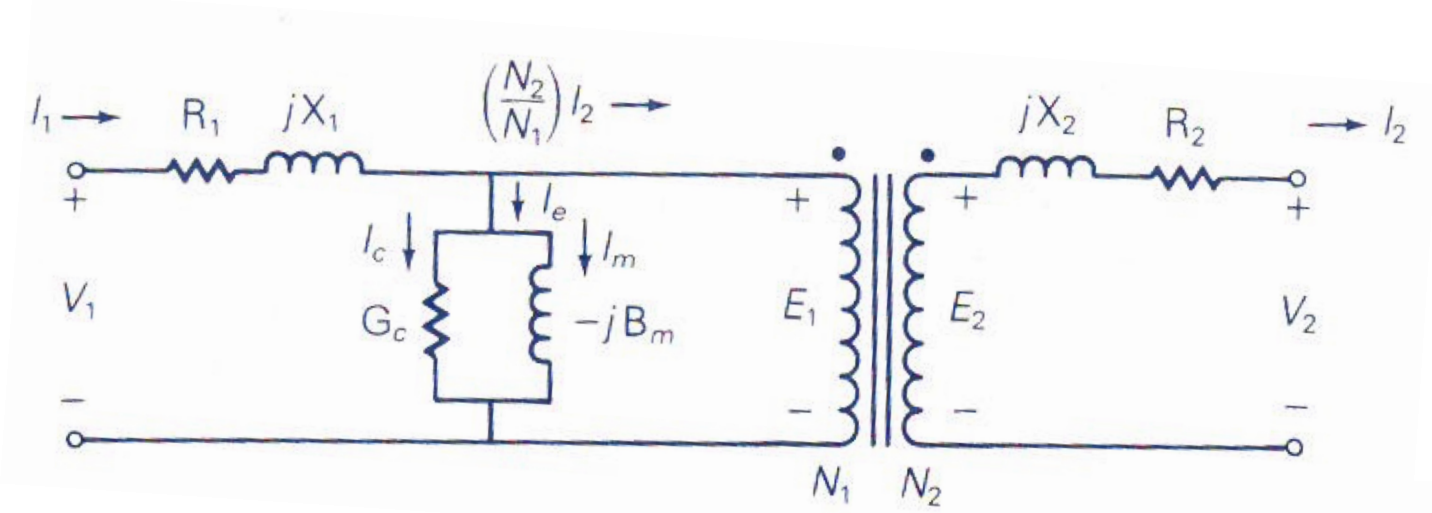
Equivalent circuit of a single-phase transformer



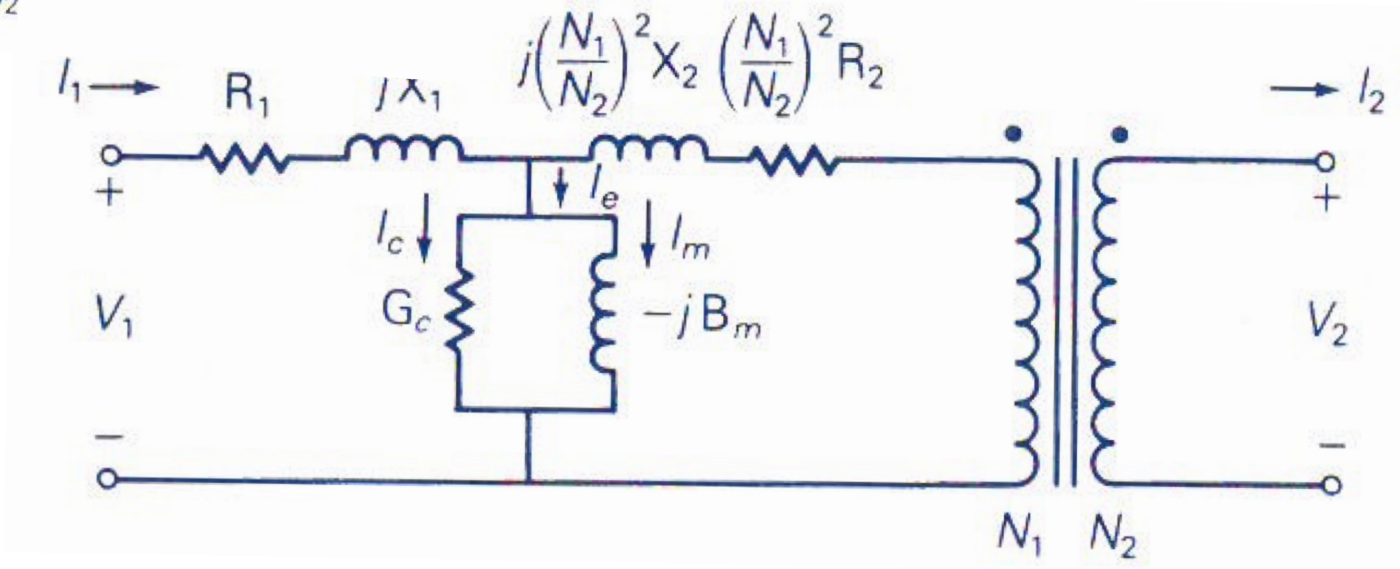
- I_c (and G_c) : core loss current
  - Hysteresis losses: cyclic variation of flux requires energy dissipated as heat
  - Eddy current losses: current perpendicular to the flux flow in the core
  - https://en.wikipedia.org/wiki/Eddy_current

- R: I^2R losses in the windings (thermal losses)
- X: leakage reactance
  - Leakage flux: links winding 1 but not winding 2, and vice versa
  - Causes a voltage drop and reactive power loss
- I_m (and B_m): magnetizing current
  - For finite core permeability μ_c, reactive power is required to magnetize the core. This reactive power loss is given by: I_m^2/B_m
- Exciting current: $I_e = I_m + I_c$

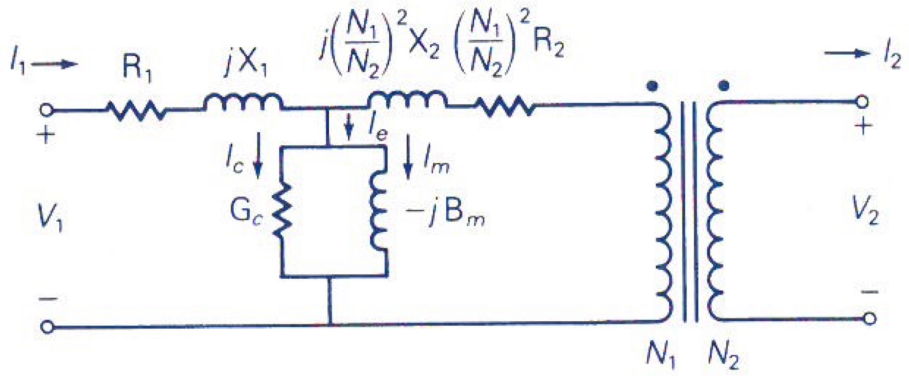
Equivalent simplified circuits



Secondary side referred to the primary side



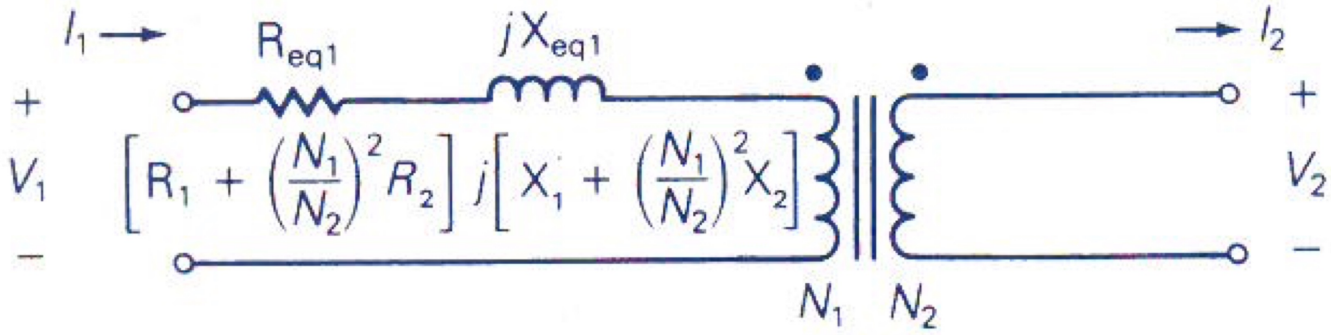
Equivalent simplified circuits



- Exciting current <5% of rated current
- Usually neglected

Secondary side referred to the primary side,

and neglecting exciting current



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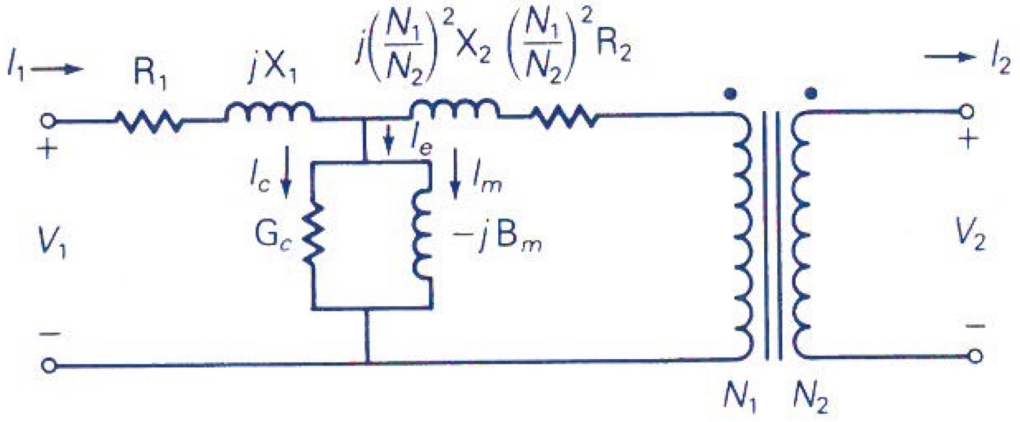
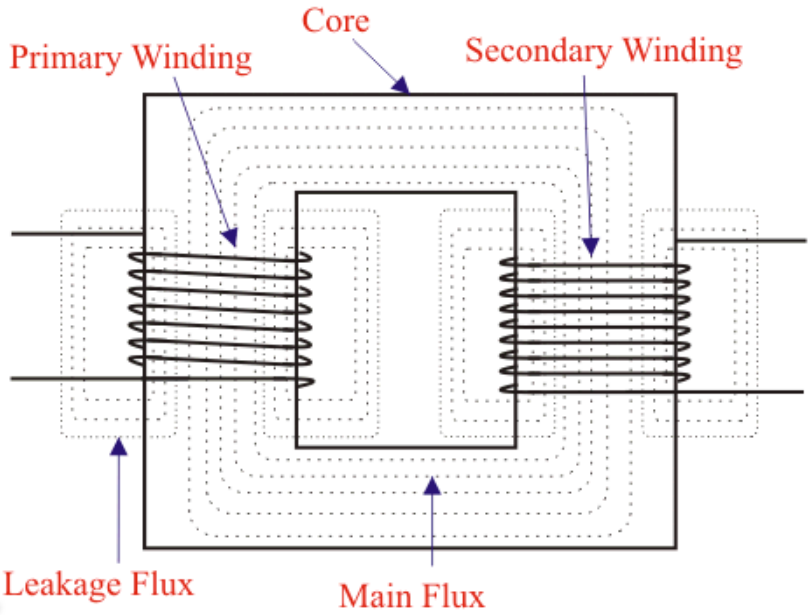




Source: https://www.electrical4u.com/

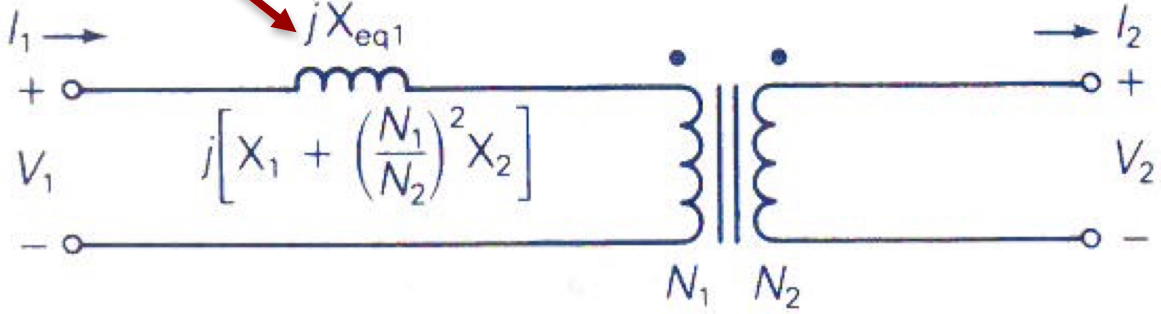
Secondary side referred to the primary side,

and neglecting exciting current

and neglecting I^2R winding loss

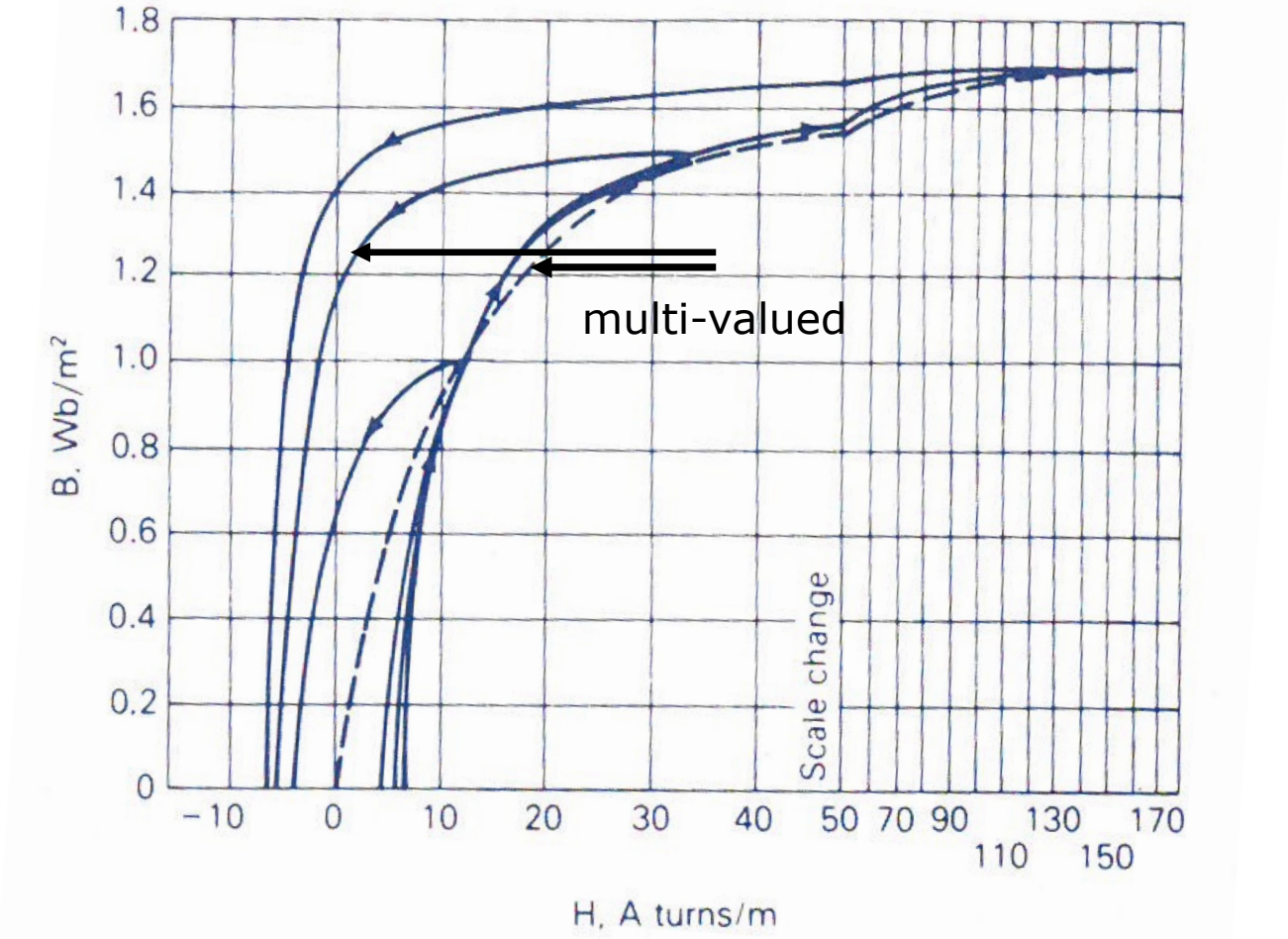
(>500 kVA: $R_{eq} << X_{eq}$)

Leakage reactance



Limitations of the equivalent circuit

- Equivalent circuit assumption:
  - μ_c is constant
  - $B_c = \mu_c H_c$
- BUT: $B_c = \mu_c(H) \cdot H_c$ -- μ_c is *not* constant
- Saturation: μ_c non-linear and multivalued (due to hysteresis)
  - large voltage → core saturates → high magnetizing current



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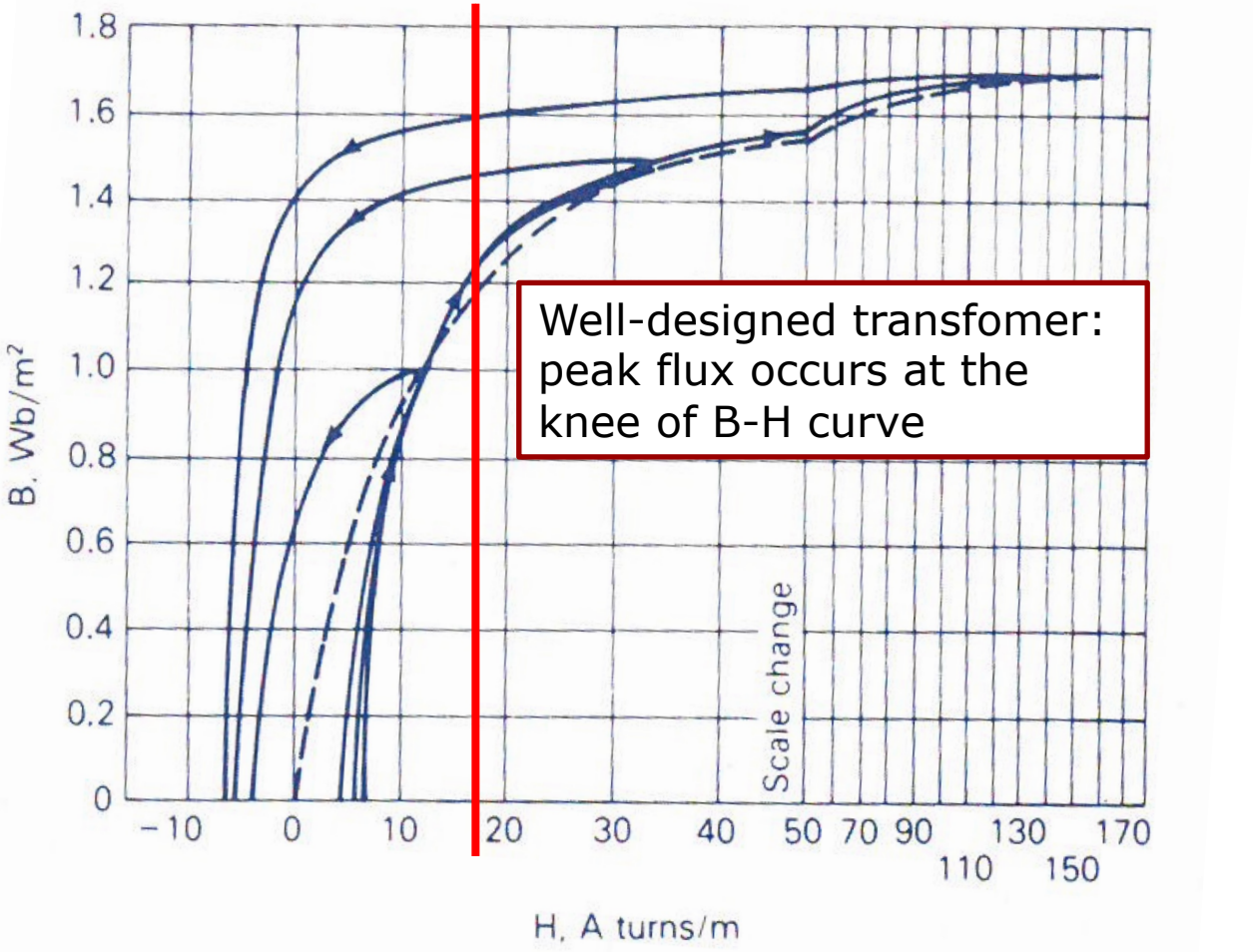
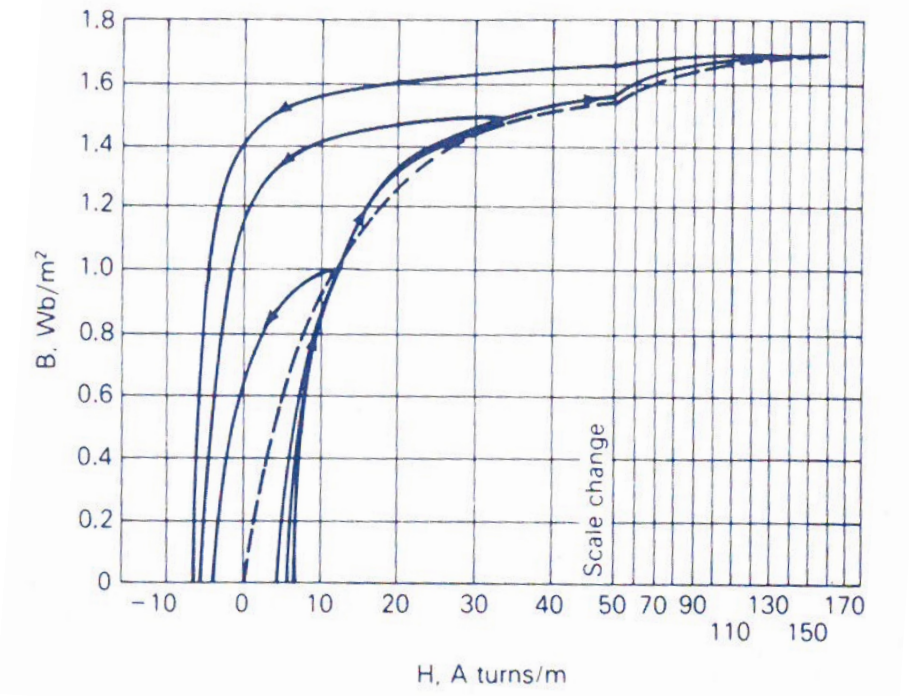


Limitations of the equivalent circuit

- Saturation: $B_c = \mu_c H_c$ -- μ_c is *not* constant
  - μ_c non-linear and multivalued (due to hysteresis)
  - large voltage → core saturates → high magnetizing current

- Inrush current: transient current much larger than rated current can flow for several cycles
  - If core already magnetized, when energizing the transformer, B(t) can reach high values
  - B(t)↑ H(t)↑↑ I_c(t) ↑↑
  - Inrush vs. short-circuit: challenge for protection

- Non-sinusoidal exciting current:
  - e(t) sinusoidal --> φ(t) sinusoidal --> B(t) sinusoidal
  - But: H(t) = [1/μ(H)] B(t) is not sinusoidal, due to the non-linear B-H relationship



- Surge phenomena: when subjected to transient overvoltages due to e.g. lightning, the capacitances of the windings have important effects (not modelled here)

Per unit system (p.u.)

- Calculation ”trick” in order to eliminate the conversion between primary and secondary side in transformers

- Simplifies transformer model and calculations

- Used almost in **any** power flow calculation

- Need to define base values for voltage, current, power, and impedance.
  - Need only to define two of four: usually voltage V_B and power S_B
  - The rest, i.e. current and impedance are calculated from the voltage and power

- Base values are always real numbers (not complex). This means:
  - p.u. values are dimensionless
  - angles of p.u. quantities are exactly the same as the angles of the actual quantities

Transforming to per unit

$$V_{pu}\angle\phi = \frac{V_{actual}\angle\phi}{V_{base}} = \frac{V_{re}+jV_{im}}{V_{base}}$$

$$S_{pu}\angle\phi = \frac{S_{actual}\angle\phi}{S_{base}} = P_{pu}+jQ_{pu}$$

$$I_{pu}\angle\phi = \frac{I_{actual}\angle\phi}{I_{base}} = \frac{I_{re}+jI_{im}}{I_{base}}$$

$$Z_{pu}\angle\phi = \frac{Z_{actual}\angle\phi}{Z_{base}} = R_{pu}+jX_{pu}$$

Phase angles in p.u. exactly the same as in the actual quantities!

Same base value for S, P, and Q !

$$P_{pu} = \frac{P_{actual}}{S_{base}} \qquad Q_{pu} = \frac{Q_{actual}}{S_{base}}$$

Same base value for Z, R, and X !

$$R_{pu} = \frac{R_{actual}}{Z_{base}} \qquad X_{pu} = \frac{X_{actual}}{Z_{base}}$$

Defining the base values

1-phase

Values we pick:

$$S_{base1\phi}$$

$$V_{base1\phi} = V_{baseLN}$$ *line-neutral voltage*

Values we calculate:

$$I_{base} = \frac{S_{base1\phi}}{V_{baseLN}}$$

$$Z_{base} = \frac{(V_{baseLN})^2}{S_{base1\phi}}$$

3-phase

Values we pick:

3-phase power $S_{base3\phi} = 3S_{base1\phi}$

line-line voltage V_{baseLL}

Values we calculate:

line-neutral voltage $V_{baseLN} = \frac{V_{baseLL}}{\sqrt{3}}$

same as in 1-phase $I_{base} = \frac{S_{base3\phi}}{\sqrt{3}V_{baseLL}} = \frac{S_{base1\phi}}{V_{baseLN}}$

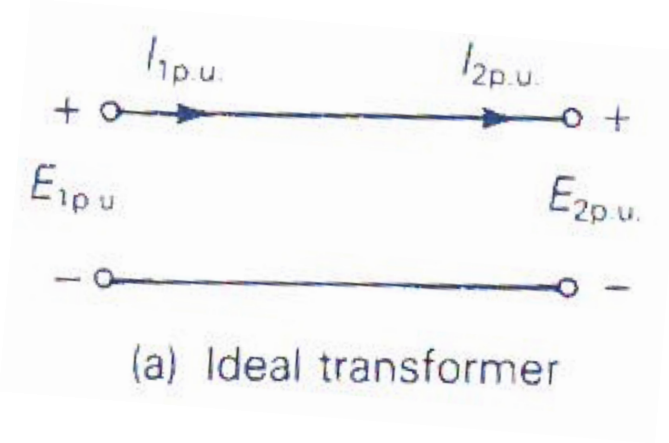
same as in 1-phase $Z_{base} = \frac{(V_{baseLL})^2}{S_{base3\phi}} = \frac{(V_{baseLN})^2}{S_{base1\phi}}$

Calculations in per unit

- For an **ideal transformer**, the primary voltage equals the secondary voltage in per unit:

$$\overline{E}_{1pu} = \overline{E}_{2pu}$$

Why?



(a) Ideal transformer

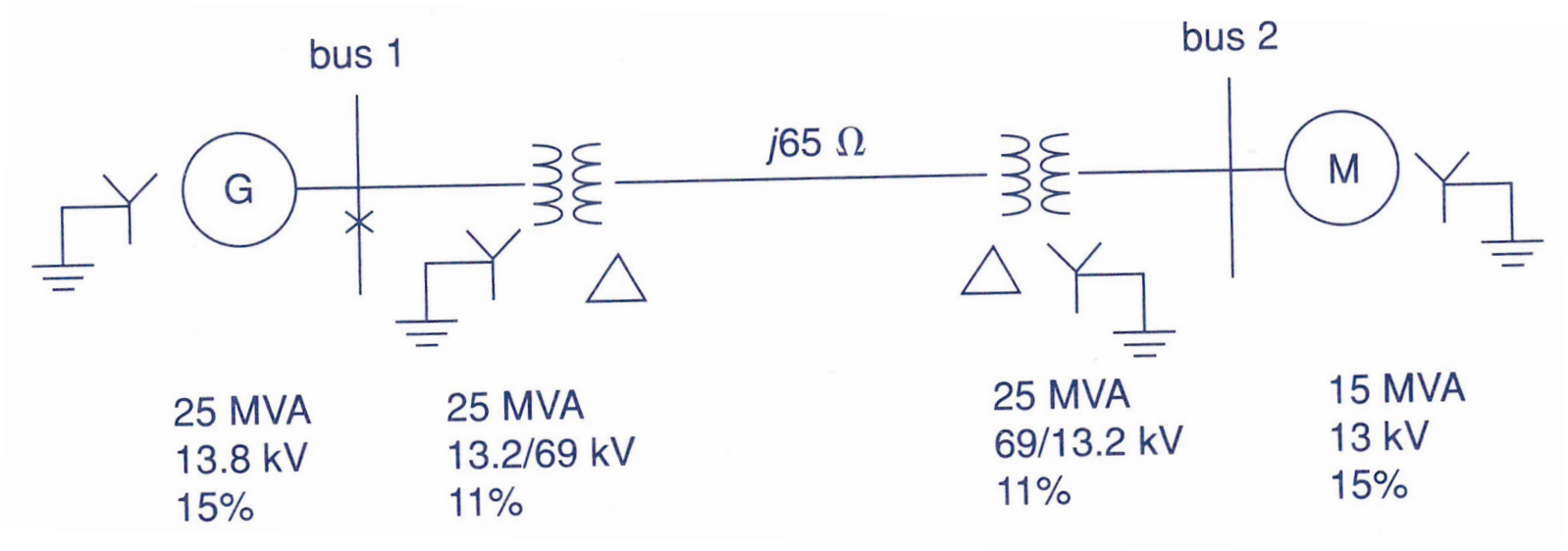
- When we do calculations in per unit, we do not need to distinguish between:
  - 1-phase and 3-phase power
  - line-line and line-neutral voltage

Both for 1-phase and 3-phase, when in p.u.:

$$\overline{S}_{pu} = \overline{V}_{pu} \cdot \overline{I}^{*}_{pu}$$

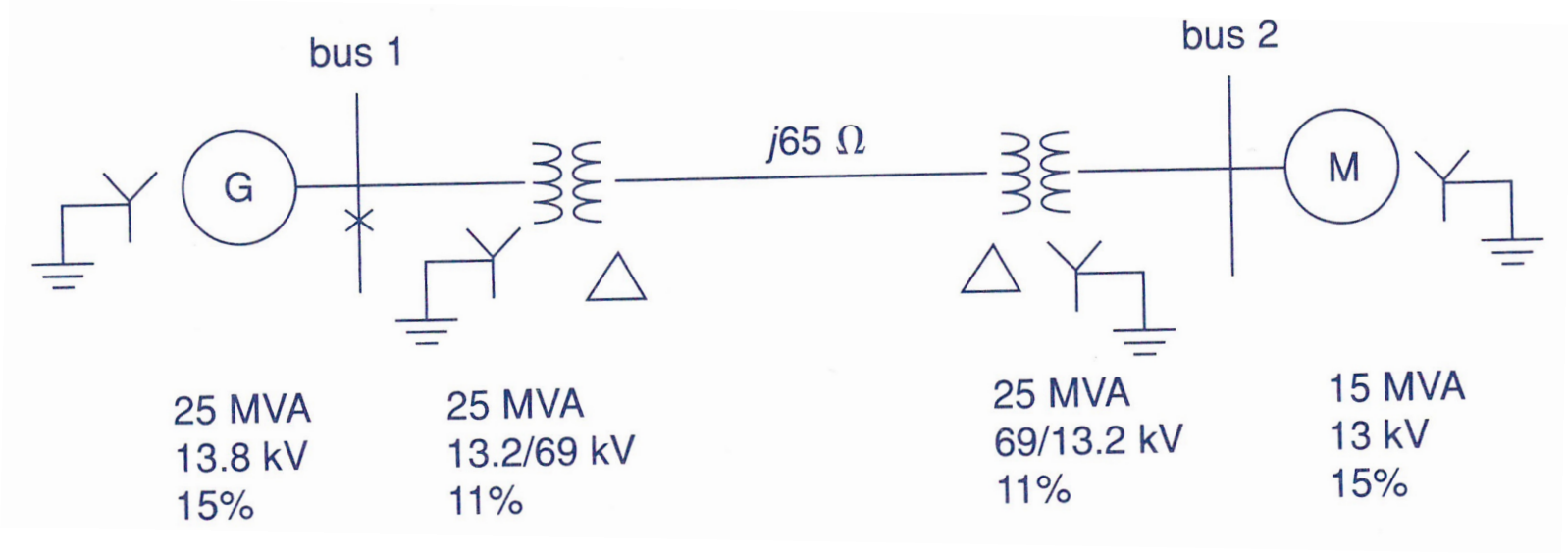
Why?

Per unit calculations



Why do we calculate in p.u.?

Per unit calculations



Why do we calculate in p.u.?

- Proper selection of base values eliminates the need for transforming from one voltage level to the other
- Then: voltage and current values in p.u. remain the same from both sides of the transformer

Per unit calculations

1. Divide our system in as many areas as $N_{TRAFO}+1$

2. Select a base voltage for each area
   - Make sure that the voltage ratio between two neighboring areas is the same as the transformer ratio between the two areas it connects

3. Select a single base MVA that will be the same for the whole system

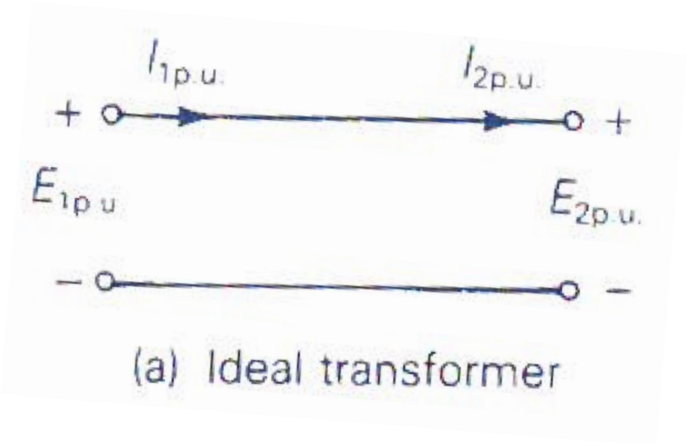
4. Calculate the base impedance in each area

5. Calculate the impedances in p.u.

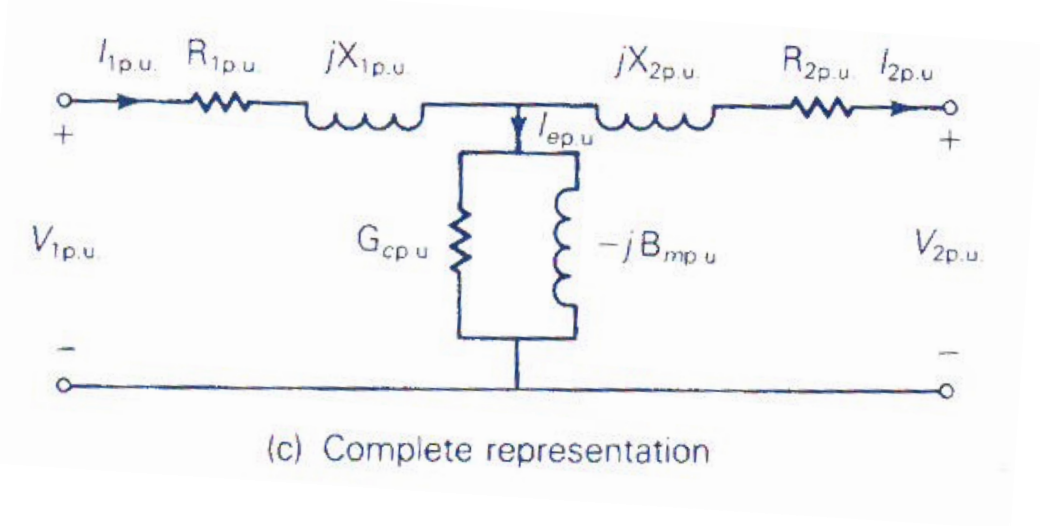
6. Calculate the voltages in p.u.

7. Calculate the power in p.u. (i.e. usually of loads or generators)

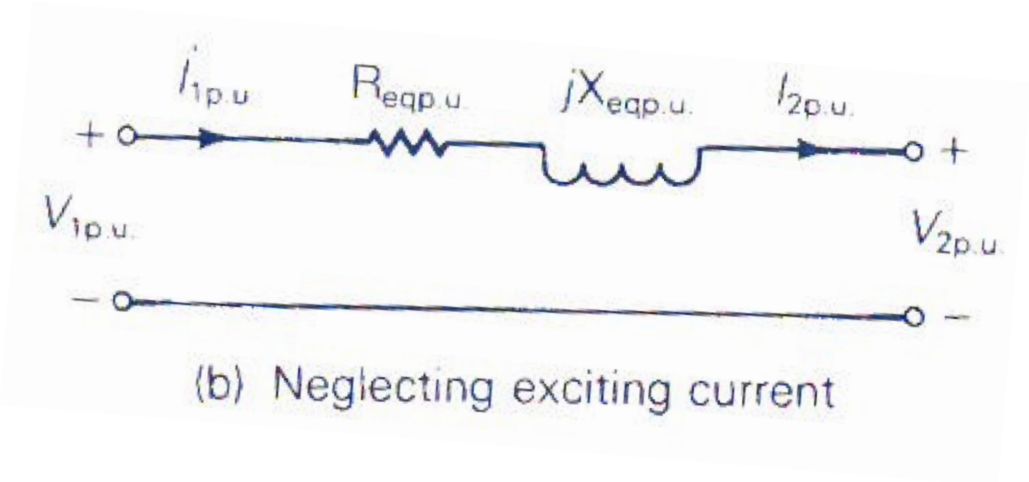
Transformers in per unit



(a) Ideal transformer

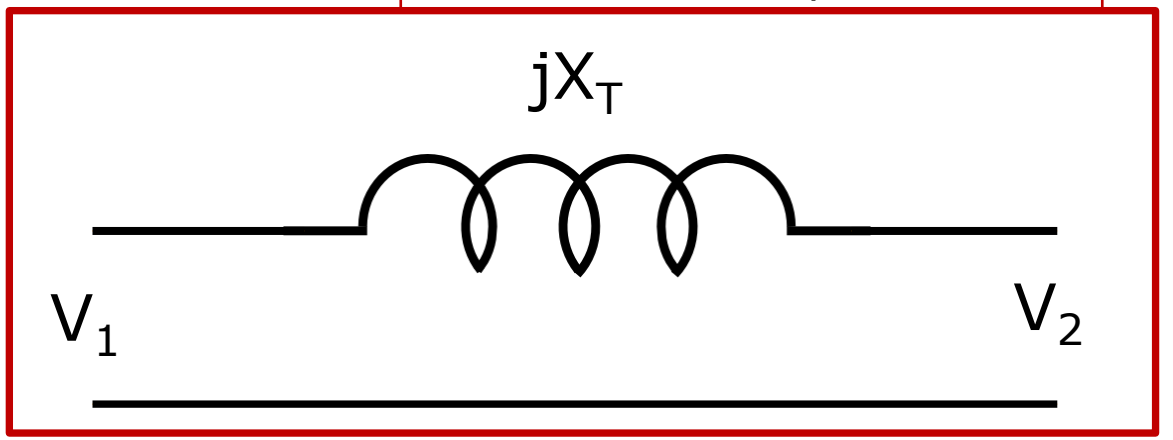


(c) Complete representation



(b) Neglecting exciting current

The most usual representation



Positive-sequence?

- The term “positive-sequence” refers to the modeling approach we use for unbalanced conditions: the symmetrical components
- Symmetrical components consist of the positive-sequence, negative-sequence, and zero-sequence
- In this course we deal only with balanced systems
- In balanced systems, the positive-sequence component corresponds to the single-phase equivalent of our three-phase system