31730: Electric Power Engineering, fundamentals

Transmission Lines

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Goals for Today

- Transmission Line Models
  - Series impedance
  - Exact model
  - **π-model of a line**

- Limits of π-model approximation

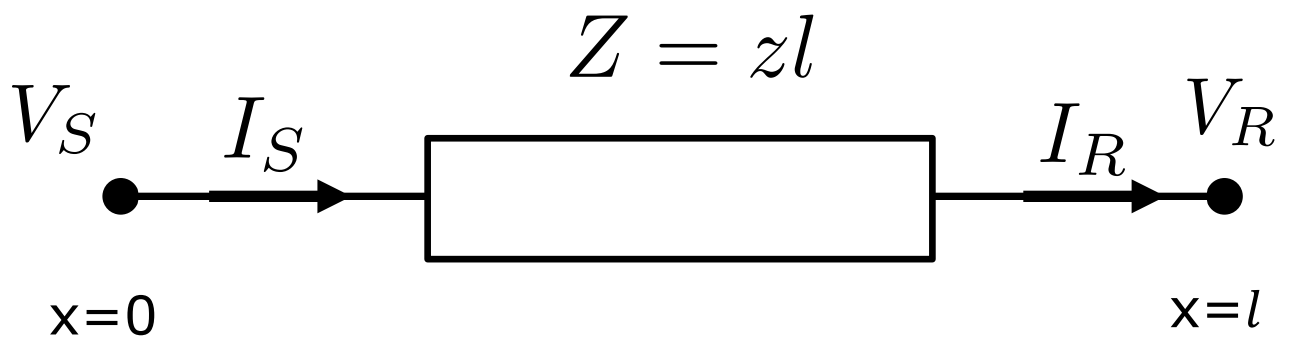
- Phasor diagrams including transmission lines

Goal: Find voltage and current at any point x along a line

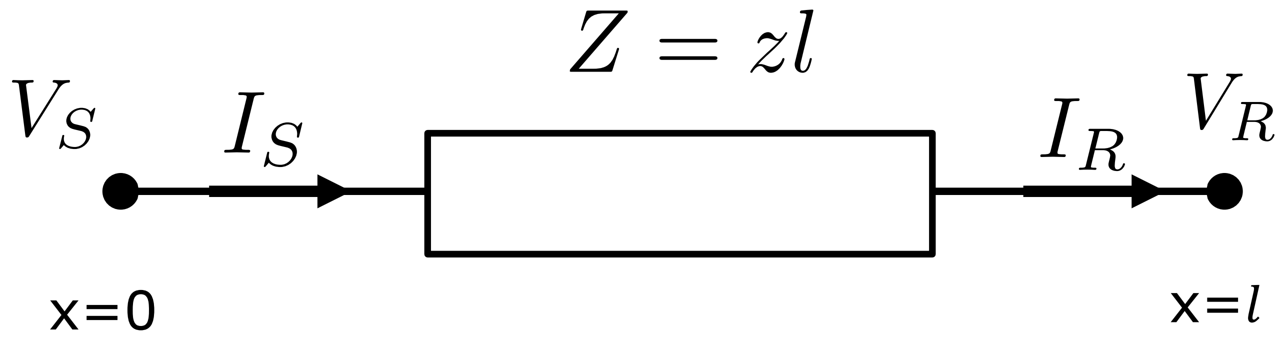
- What quantities do we need?

$$\text{Need to find} \begin{cases} V_R(x) \\ I_R(x) \end{cases}$$

Short Lines: length ≤ 25 km → Series Impedance



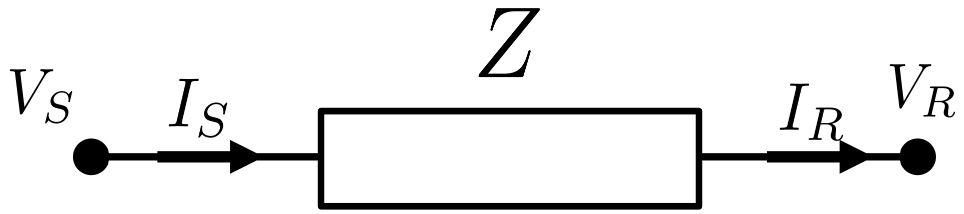
Short Lines: length ≤ 25 km → Series Impedance

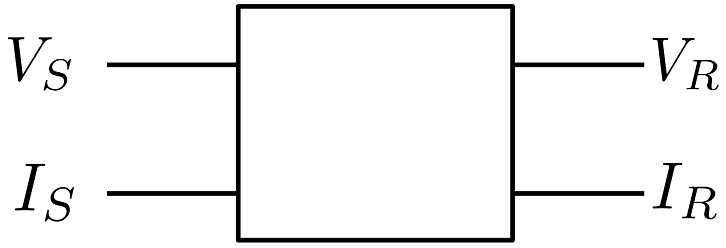


Two-port model

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Short Lines: length ≤ 25 km → Series Impedance





Find it!

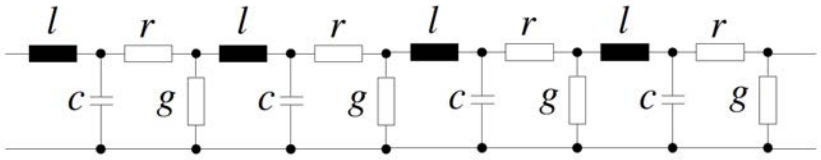
?

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Overhead line

$$z = R' + j\omega L' \qquad \Omega/m$$

$$y = G' + j\omega C' \qquad S/m$$



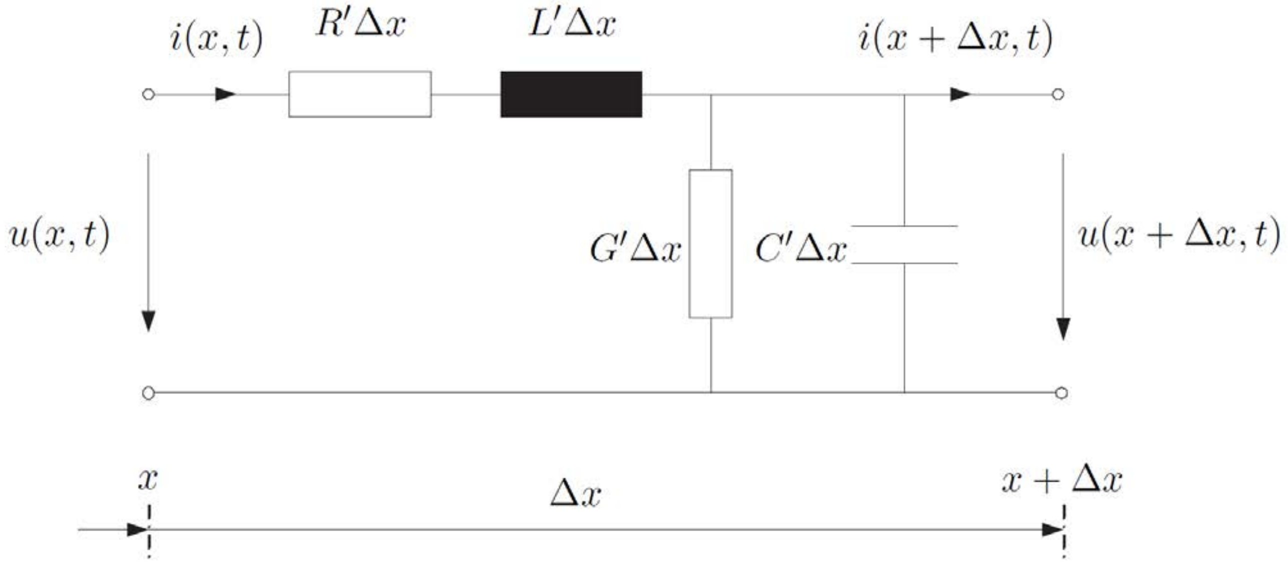


Figure taken from:
G. Andersson and C. Franck, Electric Power Systems, Lecture Notes, ETH Zurich, 2013

Medium length: length ≤250 km → π–model

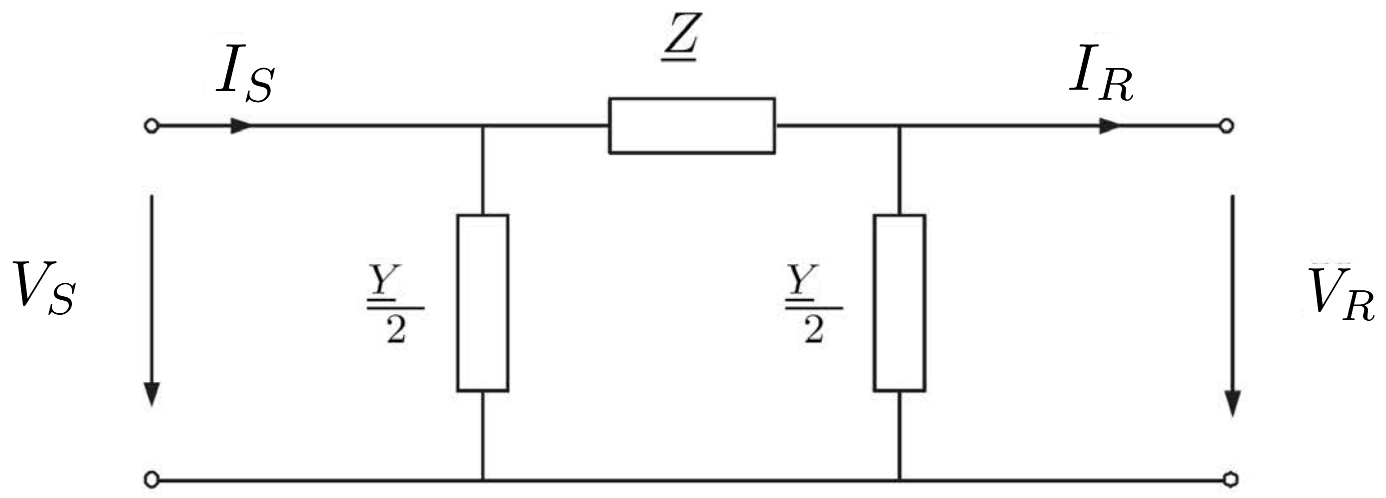


Figure 5.26. Π equivalent circuit diagram of a homogenous power line.

Figure taken from:
G. Andersson and C. Franck, Electric Power Systems,
Lecture Notes, ETH Zurich, 2013

π–model of a transmission line

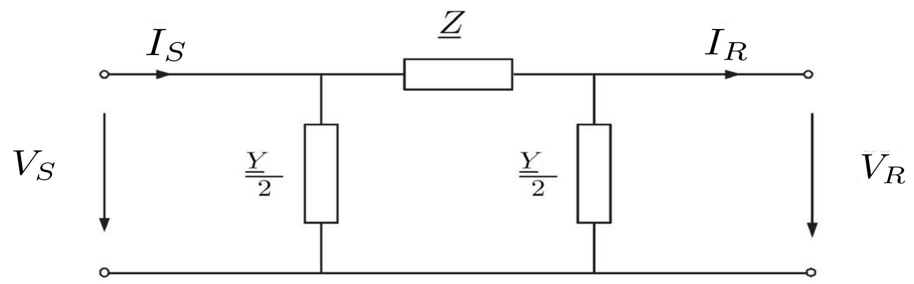


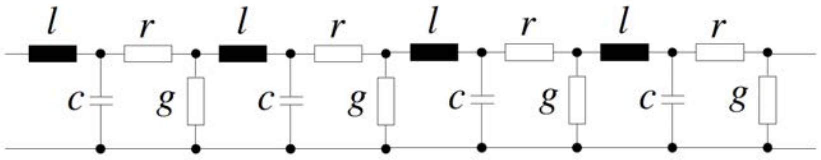
Figure 5.26. Π equivalent circuit diagram of a homogenous power line.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{YZ}{2} & Z \\ Y\left(1 + \frac{YZ}{4}\right) & 1 + \frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Overhead line

$$z = R' + j\omega L' \qquad \Omega/m$$

$$y = G' + j\omega C' \qquad S/m$$



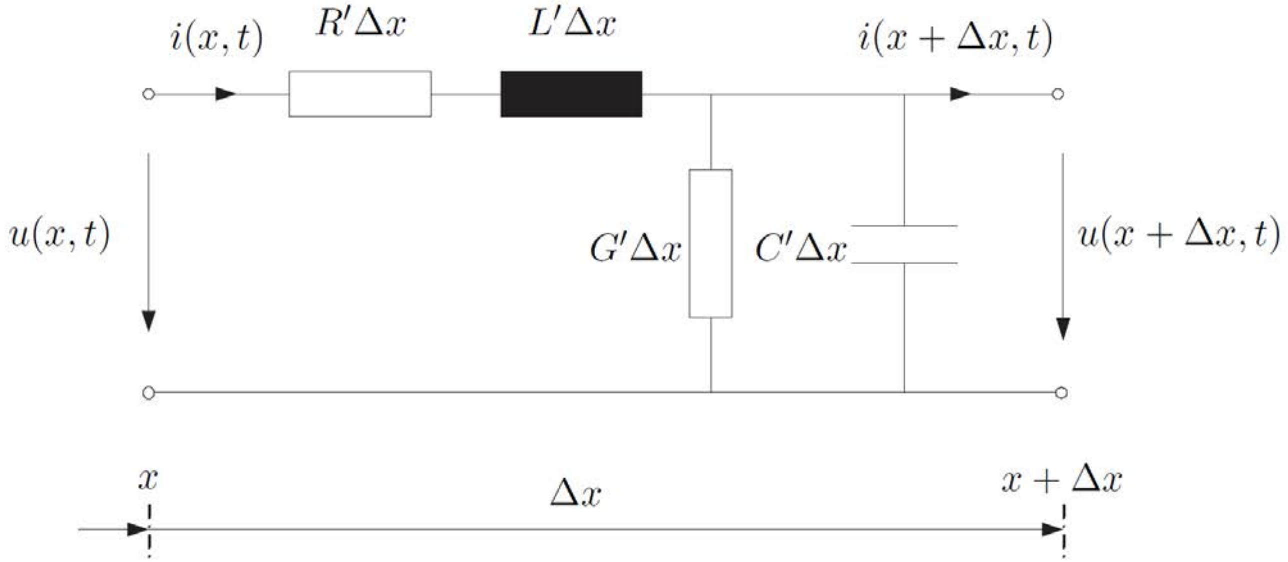


Figure taken from:
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Exact Equations: Valid for any line length. <u>Must</u> use for length > 250 km

$$z = R' + j\omega L' \qquad \Omega/m$$

$$y = G' + j\omega C' \qquad S/m$$

$$\gamma = \sqrt{zy}$$

$$Z_c = \sqrt{\frac{z}{y}}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{1}{Z_c} \sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

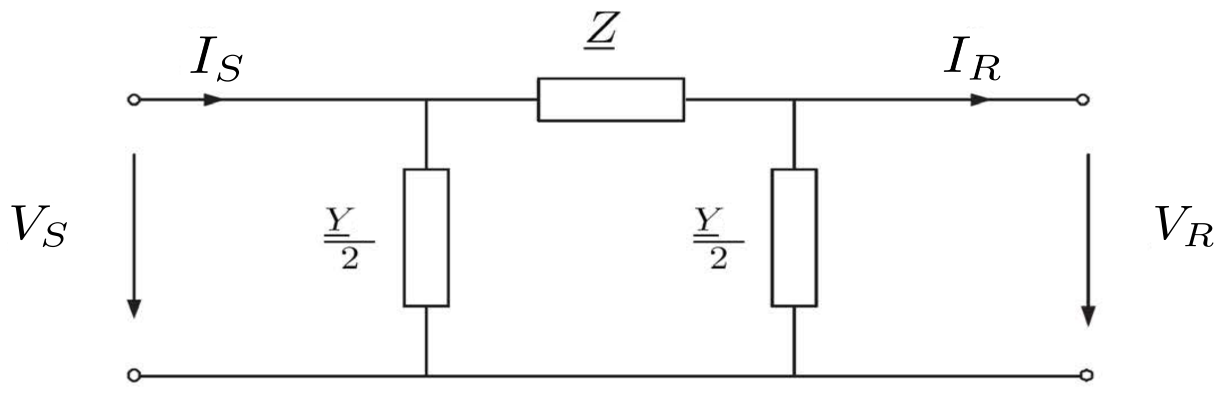
Transforming the exact equations to an equivalent π–model

- How much is Z´and Y´?

$$\begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{1}{Z_c}\sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} = \begin{bmatrix} 1 + \frac{Y'Z'}{2} & Z' \\ Y\left(1 + \frac{Y'Z'}{4}\right) & 1 + \frac{Y'Z'}{2} \end{bmatrix}$$

$$\gamma = \sqrt{zy}$$

$$Z_c = \sqrt{\frac{z}{y}}$$



Exact Equations

$$Z' = Z_c \sinh(\gamma l)$$

$$\frac{Y'}{2} = \frac{1}{Z_c} \tanh\left(\frac{\gamma l}{2}\right)$$

Model always valid

π–model

for $\gamma l << 1$

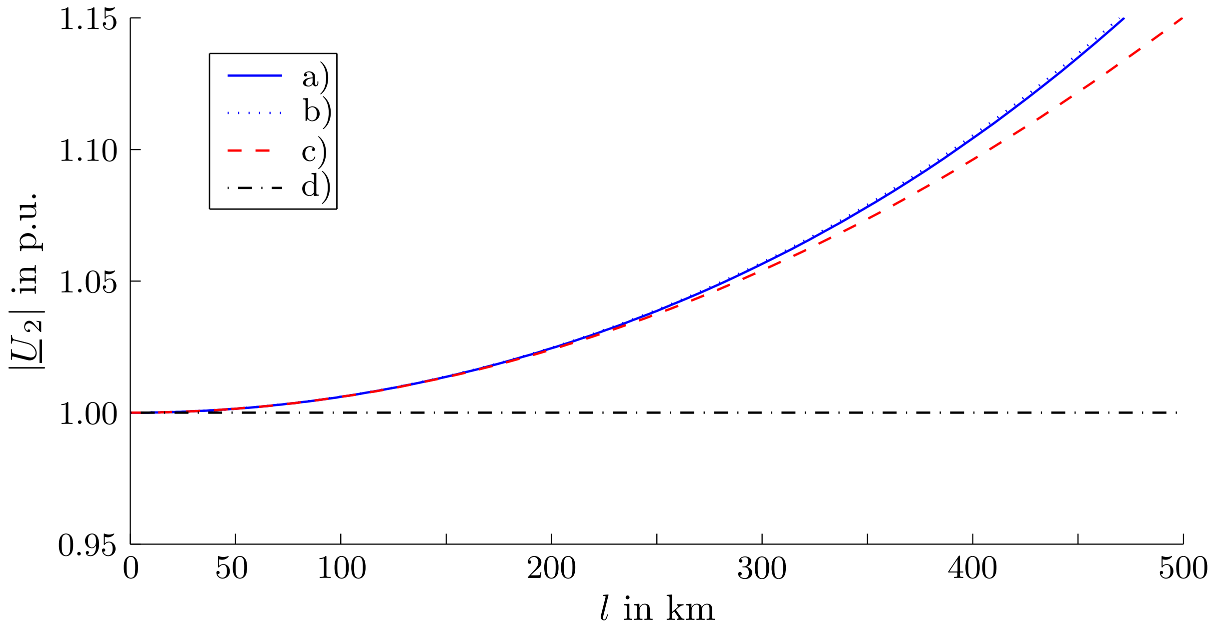
$$Z' = zl = Z$$

$$Y' = yl = Y$$

Valid when $\gamma l << 1$

This is usually the case for length ≤ 250 km

Accuracy of different Line Model Approximations



Absolute value of the no-load voltage at the end of the line computed with different models

a) exact equations: 5.2.33-5.2.36

b) wave equation for lossless line: 5.4.10-5.4.12

c) Π-model: 5.1.14-5.1.17

d) Series impedance: 5.1.7-5.1.10

Check Table 5.1 and know when to use each model

length < 25 km:	series impedance
25 km<length<250 km:	Π-model
Length > 250 km:	exact equations

Figure taken from:
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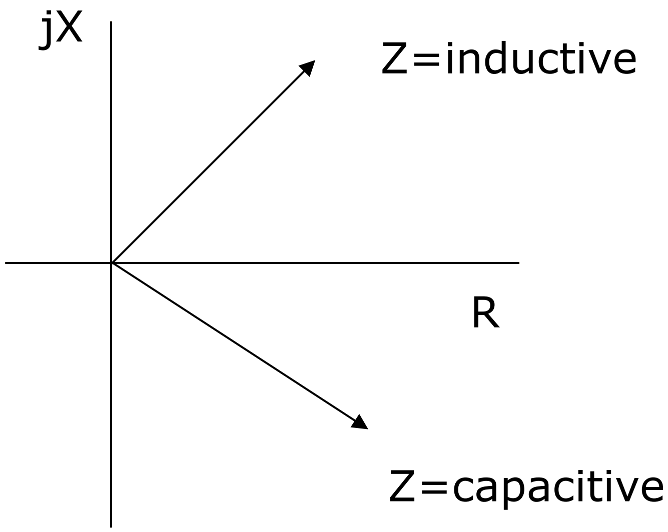
Identify the inductive and capacitive quadrants

$Z = R+jX$

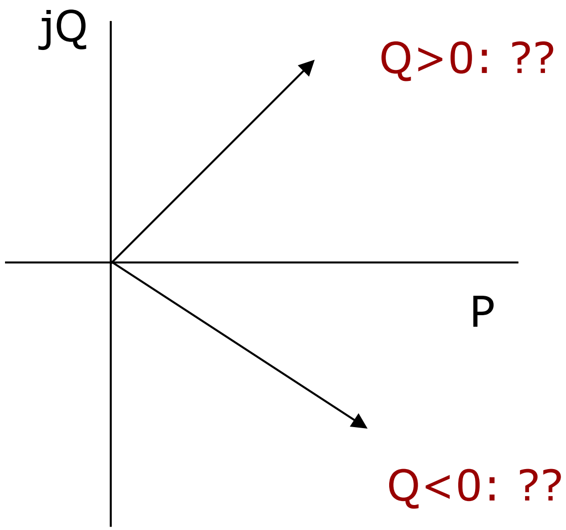
X positive → inductive



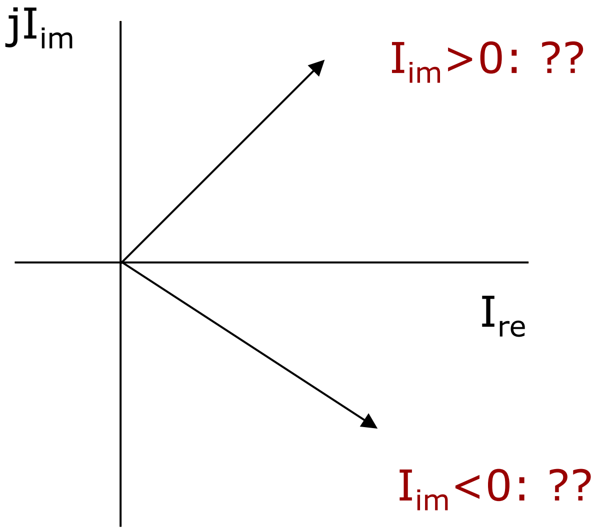
X negative → capacitive

5

$S_{load} = P+jQ$



$I_{load} = I_{re}+jI_{im}$



Phasor Diagrams

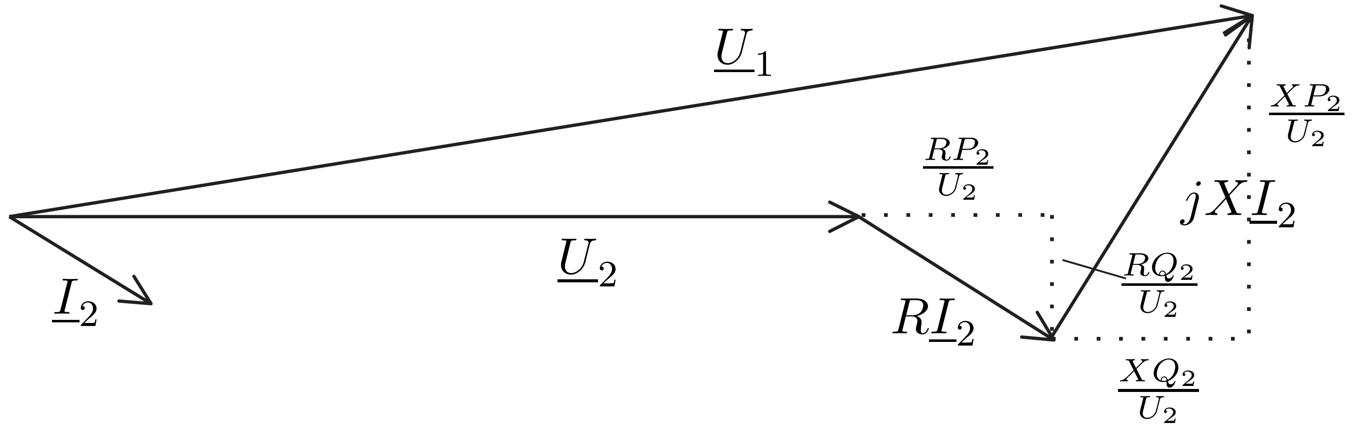


Figure 6.6. Relation between the phasors $\underline{U}_1$ and $\underline{U}_2$.

How will the phasor diagram look like for a capacitive load?

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Video

https://www.youtube.com/watch?v=tGQPZzFbLPE