31730: Electric Power Engineering, fundamentals

# Transmission Lines

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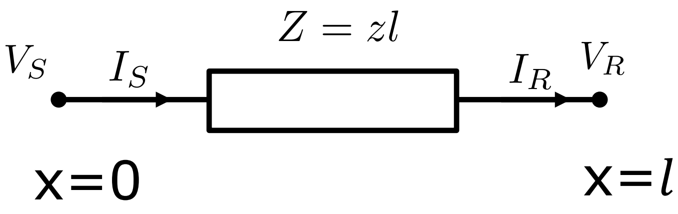
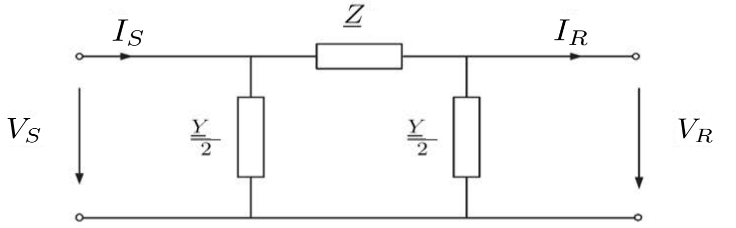
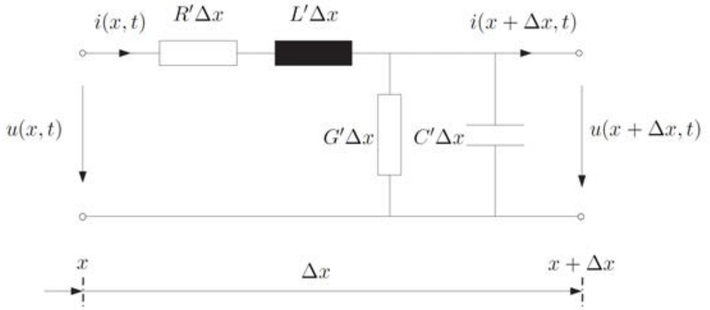
# Summary from last time




$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

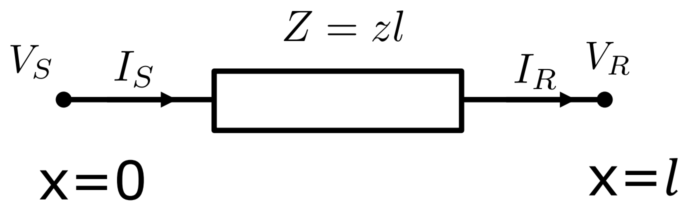
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{YZ}{2} & Z \\ Y\left(1 + \frac{YZ}{4}\right) & 1 + \frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{1}{Z_c} \sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$Y = 0$$

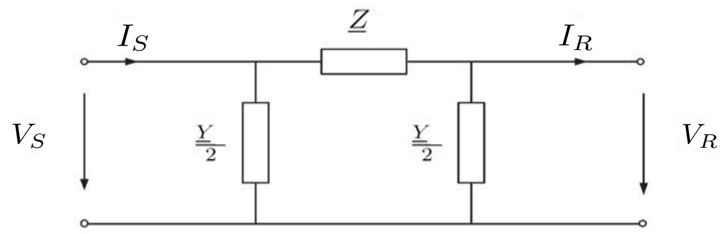
$$\gamma l << 1$$

# Summary from last time



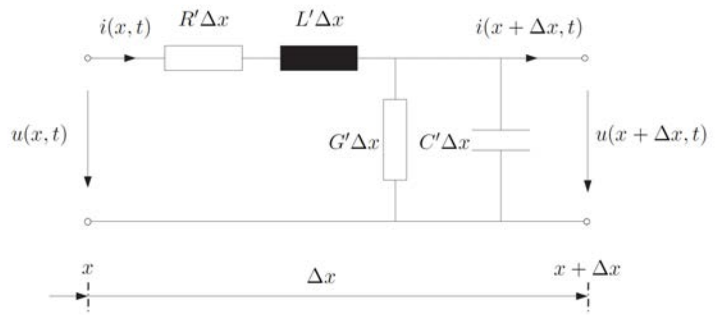

$l \leq 25$ km

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$




$l \leq 250$ km

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{YZ}{2} & Z \\ Y\left(1 + \frac{YZ}{4}\right) & 1 + \frac{YZ}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$




$l > 250$ km

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{1}{Z_c} \sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$Y = 0$

$\gamma l << 1$

# Questions from last time

- What is the meaning of $\gamma l$ ?
  - It has to do with the propagation of the voltage wave. Even if no impedance is assumed the voltage is a traveling wave and as a result at one point x1 it is going to be different from a voltage at another point x2
  - The Ferranti effect has to do with these changes of voltage in space

# Goals for Today

- Lossless line
- Surge impedance
- Surge impedance loading
- Voltage profiles
- Steady-state stability limit
- Maximum Power Flow
- Line Loadability

# Surge impedance and propagation constant for lossless lines

$$z = R' + j\omega L' \qquad \Omega/m$$

$$y = G' + j\omega C' \qquad S/m$$

$$Z_c = \sqrt{\frac{z}{y}}$$

$$\gamma = \sqrt{zy}$$

$$\sinh(y) = \frac{e^y - e^{-y}}{2}$$

$$\cosh(y) = \frac{e^y + e^{-y}}{2}$$

- For lossless lines
  - R′ = 0
  - G′ = 0

1. How much is Zc and γ?
2. How much are the ABCD parameters?

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ \frac{1}{Z_c} \sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

# ABCD Parameters

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cos(\beta l) & Z_c \sin(\beta l) \\ \frac{1}{Z_c} \sin(\beta l) & \cos(\beta l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

- For a lossless line:
  - The surge impedance is pure resistive
  - The propagation constant is pure imaginary

- For βl < π, the equivalent π-circuit is also lossless
  - βl < π: for l<3000 km for 50 Hz

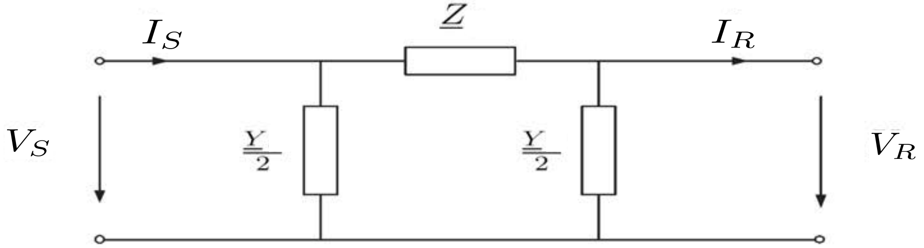



**Figure 5.26.** Π equivalent circuit diagram of a homogenous power line.

# Wavelength

- Wavelength: distance required to change the phase of voltage or current by 2π

$$V(x) = \cos(\beta x) V_R + j Z_c \sin(\beta x) I_R$$

$$I(x) = \frac{j \sin(\beta x)}{Z_c} V_R + \cos(\beta x) I_R$$

$$\lambda = \frac{2\pi}{\beta} \qquad f\lambda = \boxed{\frac{1}{\sqrt{LC}}}$$

- Velocity of propagation of voltage and current along a lossless line
- For overhead lines it is equal to the light speed, i.e. 3*10$^8$ m/s

How much is λ?

# Wavelength

- Wavelength: distance required to change the phase of voltage or current by 2π

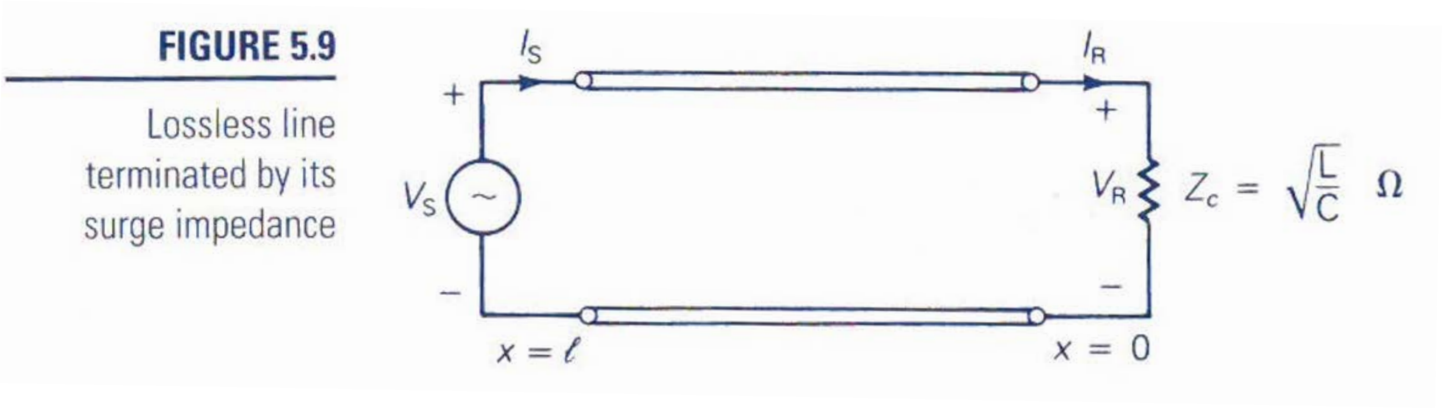
$$V(x) = \cos(\beta x)V_R + jZ_c \sin(\beta x)I_R$$

$$I(x) = \frac{j\sin(\beta x)}{Z_c}V_R + \cos(\beta x)I_R$$

$$\lambda = \frac{2\pi}{\beta} \qquad f\lambda = \frac{1}{\sqrt{LC}}$$

$$\lambda = 6000 \text{ km for } 50 \text{ Hz}$$

# Surge Impedance




How much is

1. V(x) ?
2. I(x) ?
3. S(x) ?

$$I_R = \frac{V_R}{Z_c}$$

*Figure taken from:*
*J. Glover, T. Overbye, M. Sarma, Power System Analysis and Design, Cengage Learning, Sixth Edition, 2016*

# Voltage Profiles

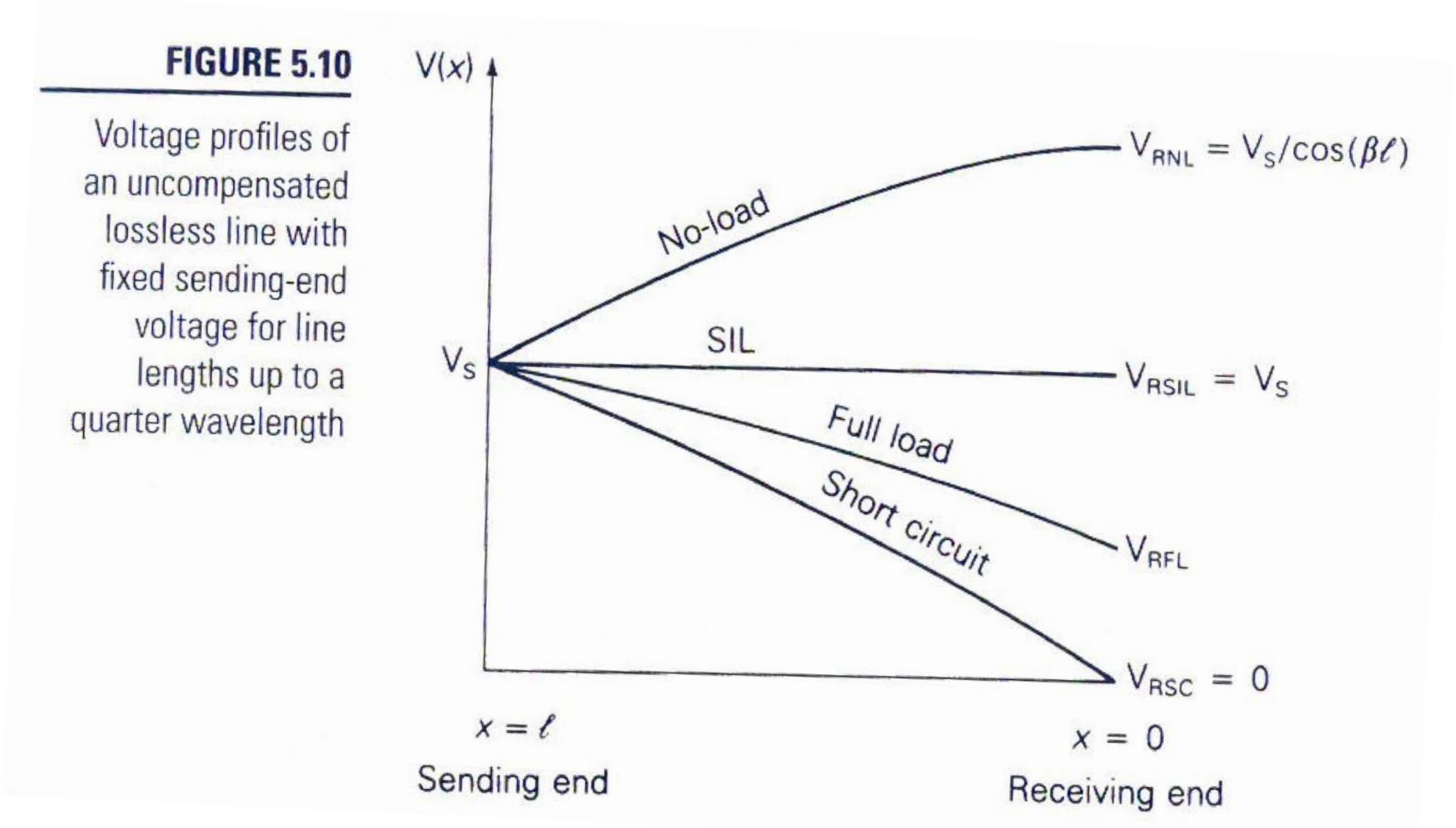



**FIGURE 5.10** Voltage profiles of an uncompensated lossless line with fixed sending-end voltage for line lengths up to a quarter wavelength

*Figure taken from:*
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# Steady-state stability limit

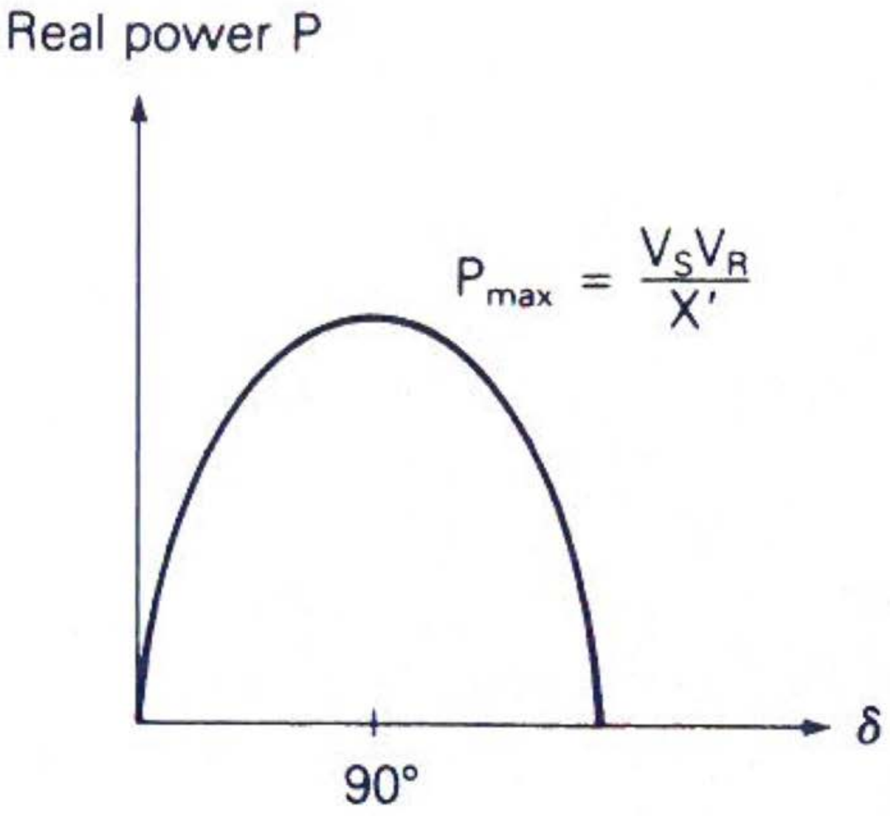
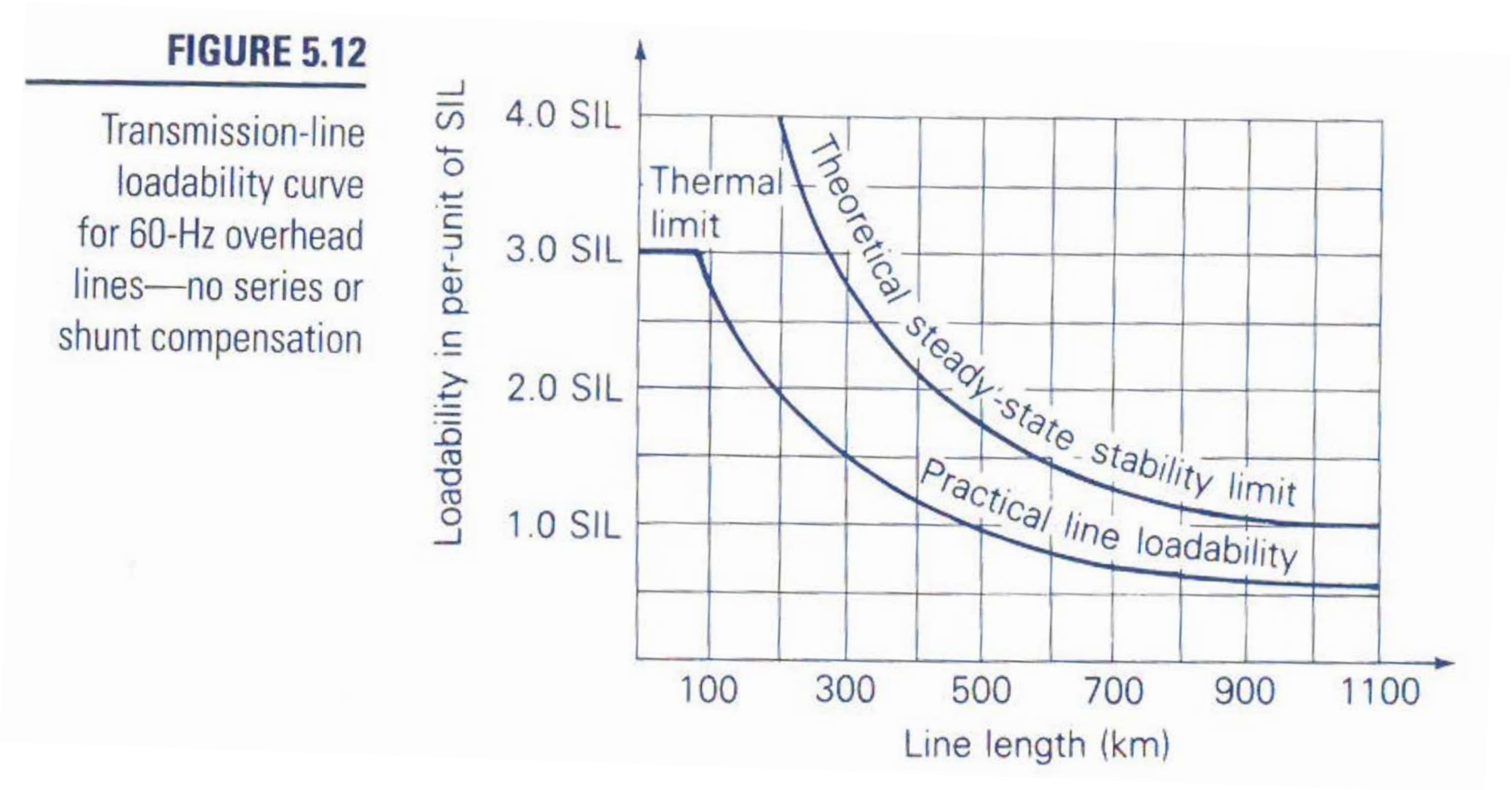



**FIGURE 5.11**

Real power delivered by a lossless line versus voltage angle across the line

*Figure taken from:*
*J. Glover, T. Overbye, M. Sarma, Power System Analysis and Design, Cengage Learning, Sixth Edition, 2016*

# Line Loadability

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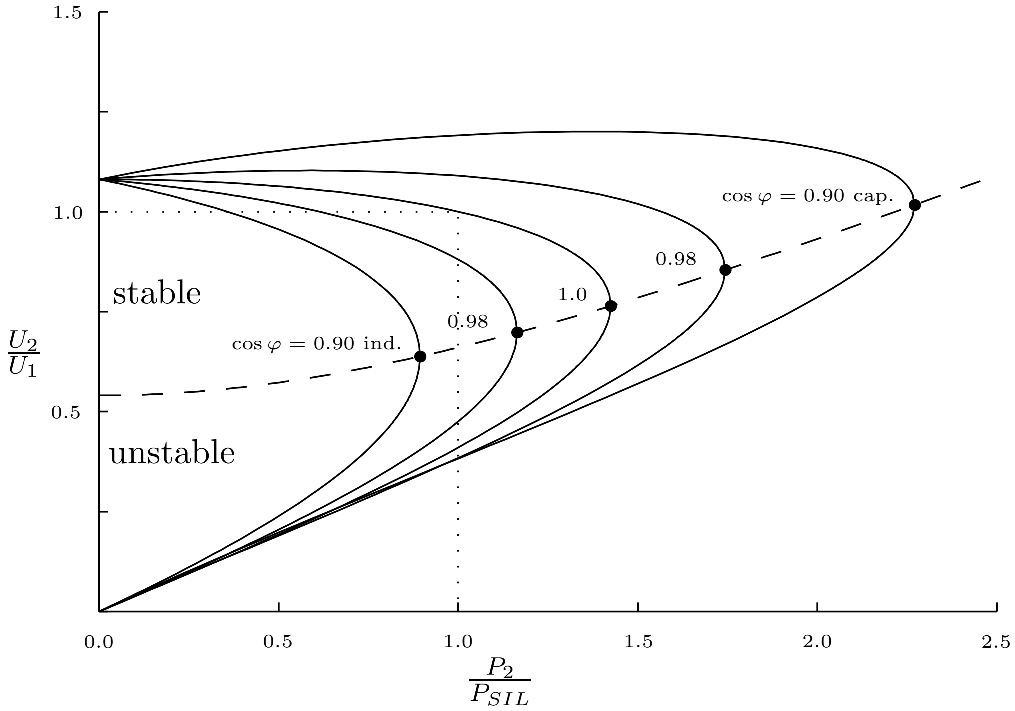


**Figure 6.10.** Voltage at the end of the line as a function of the delivered active power for different line loadings. $U_1$ is kept constant at 1.0 pu.

*Figure taken from:*
*G. Andersson and C. Franck, Electric Power Systems, Lecture Notes, ETH Zurich, 2013*