Optimization in modern power systems

Lecture 9: QP DC-OPF

Spyros Chatzivasileiadis

DTU Electrical Engineering
Department of Electrical Engineering

The Goals for Today!

- Mid-term Feedback
- Review of Day 8
- Questions and Clarifications on Assignments
- Example: Dual of DC-OPF
- Quadratic Programming and DC-OPF
- Active Power Losses in AC-OPF
- N-1 security criterion (if there is time)

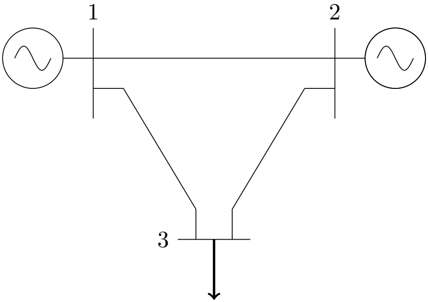
Reviewing Day 8 in Groups!

- For 10 minutes discuss with the person sitting next to you about:
  - Three main points we discussed in yesterday's lecture
  - One topic or concept that is not so clear to you and you would like to hear again about it

Points you would like to discuss?

Questions about the Assignments?

Question: What is the dual of the DC-OPF?



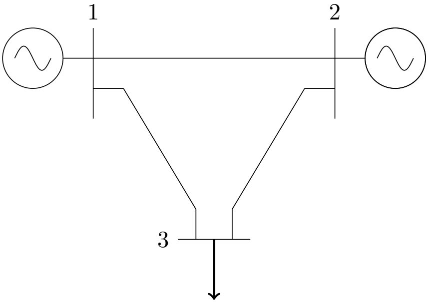
$$\min c_1 P_{G1} + c_2 P_{G2}$$

subject to:

$$B\theta = P_G - P_L$$
$$P_G \geq 0$$

- no line flow constraints

Question: What is the dual of the DC-OPF?



$$\min\ c_1 P_{G1} + c_2 P_{G2}$$

subject to:

$$B\theta = P_G - P_L$$
$$P_G \geq 0$$

- no line flow constraints

Dual Problem

$$\max\ -b^T \nu$$

subject to $A^T \nu + c \geq 0$

Quadratic Programming

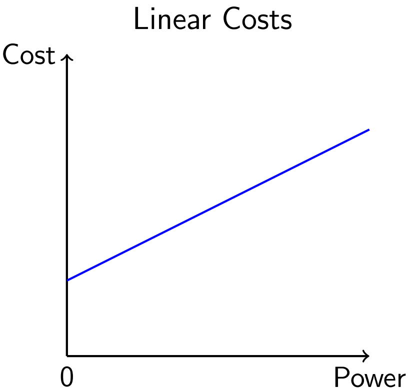
$$\min \frac{1}{2}x^T Q x + c^T x \qquad (1)$$

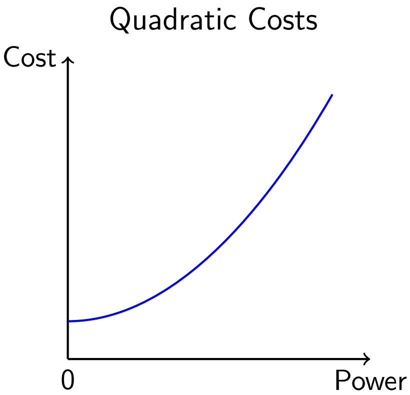
subject to:

$$\begin{aligned} g_i \cdot x \leq h_i, &\quad i = 1, \dots, m \\ a_i \cdot x = b_i, &\quad i = 1, \dots, p \end{aligned} \qquad (2)$$

- The only difference between the LP and the QP is in the objective function
- QP is not necessarily convex!
- QP convex $\Leftrightarrow Q \succeq 0$, i.e. positive semidefinite

Linear vs. Quadratic Costs in the OPF





- Linear costs *usually* represent price bids ⇒ Markets
- e.g. bid 80 \$/MWh for the next 1 hour

- Quadratic costs *usually* approximate fuel costs (and other power plant costs) ⇒ verticallly integrated utilities that wish to minimize costs

DC-OPF with Quadratic Costs

$$\min \sum_i c_{2,i} P_{G_i}^2 + c_{1,i} P_{G_i} + c_{0,i}$$

subject to:

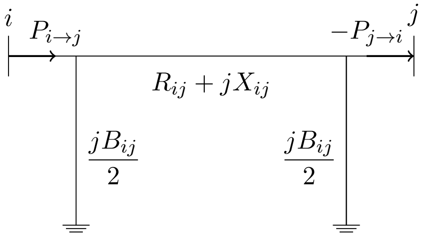
$$P_{G_i}^{min} \leq P_{G_i} \leq P_{G_i}^{max}$$

$$\mathbf{B} \cdot \theta = \mathbf{P_G} - \mathbf{P_D}$$

$$P_{ij,max} \leq \frac{1}{x_{ij}} (\theta_i - \theta_j) \leq P_{ij,max}$$

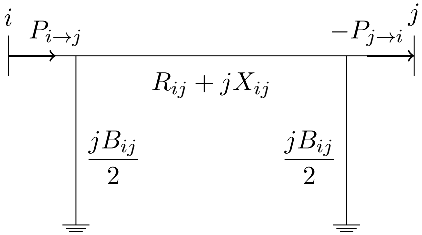
- A DC-OPF with quadratic costs is a convex problem
- $\frac{1}{2} x^T Q x + c^T x$: How does Q look like in a 'QP' DC-OPF?

Active Power Losses in AC-OPF



π-model of the line

Active Power Losses in AC-OPF



π-model of the line

- Losses = "P leaving node i" − "P arriving at node j"
- P leaving node i: $P_{i\to j}$
- P arriving at node j: $-P_{j\to i}$

$$P_{\text{losses}} = P_{i\to j} + P_{j\to i}$$