

Optimization in modern power systems

Lecture 9: QP DC-OPF

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A decorative graphic of various mathematical symbols and formulas in different colors (blue, orange, pink, purple, red) is overlaid on the slide. The symbols include: a large integral sign with 'a' and 'b' as limits; a Greek letter epsilon; a large theta; a plus sign; a square root with '17' inside; a Greek letter omega; a function 'f'; a delta symbol; an equals sign; 'e^{i\pi} = -1'; a curly brace containing the number '2.7182818284'; a Greek letter lambda; a Greek letter chi with a superscript '2'; a large sigma; a greater-than sign; a comma; a tilde symbol; and an exclamation mark.

$$f(x+\Delta x) = \sum_{i=0}^{\infty} \frac{(\Delta x)^i}{i!} f^{(i)}(x)$$

The Goals for Today!

- Mid-term Feedback
- Review of Day 8
- Questions and Clarifications on Assignments
- Example: Dual of DC-OPF
- Quadratic Programming and DC-OPF
- Active Power Losses in AC-OPF
- N-1 security criterion (if there is time)

Reviewing Day 8 in Groups!

- For 10 minutes discuss with the person sitting next to you about:
 - Three main points we discussed in yesterday's lecture
 - One topic or concept that is not so clear to you and you would like to hear again about it



Points you would like to discuss?

Questions about the Assignments?

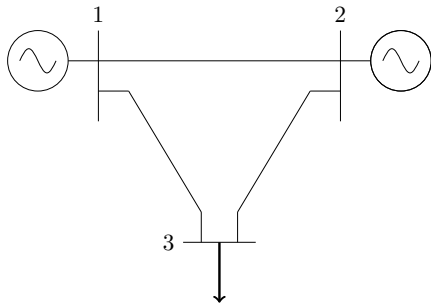
Question: What is the dual of the DC-OPF?

$$\min c_1 P_{G1} + c_2 P_{G2}$$

subject to:

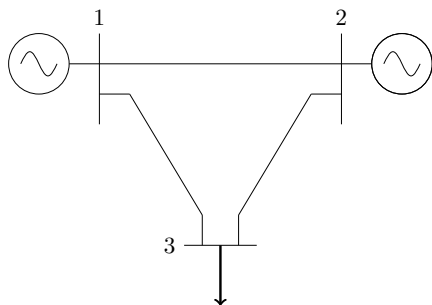
$$B\theta = P_G - P_L$$

$$P_G \geq 0$$



- no line flow constraints

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Dual Problem

$$\max -b^T \nu$$

subject to $A^T \nu + c \geq 0$

Quadratic Programming

$$\min \frac{1}{2} x^T Q x + c^T x \quad (1)$$

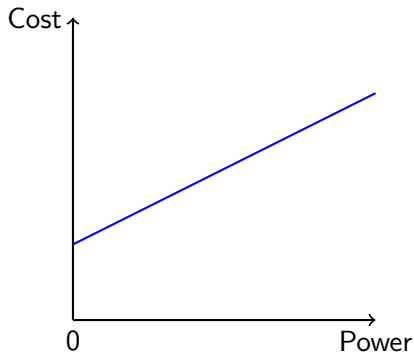
subject to:

$$\begin{aligned} g_i \cdot x &\leq h_i, & i = 1, \dots, m \\ a_i \cdot x &= b_i, & i = 1, \dots, p \end{aligned} \quad (2)$$

- The only difference between the LP and the QP is in the objective function
- QP is not necessarily convex!
- QP convex $\Leftrightarrow Q \succeq 0$, i.e. positive semidefinite

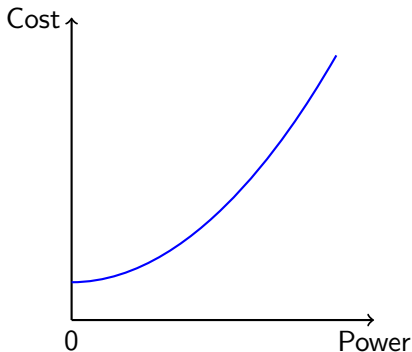
Linear vs. Quadratic Costs in the OPF

Linear Costs



- Linear costs *usually* represent price bids \Rightarrow Markets
- e.g. bid 80 \$/MWh for the next 1 hour

Quadratic Costs



- Quadratic costs *usually* approximate fuel costs (and other power plant costs) \Rightarrow vertically integrated utilities that wish to minimize costs

DC-OPF with Quadratic Costs

$$\min \sum_i c_{2,i} P_{G_i}^2 + c_{1,i} P_{G_i} + c_{0,i}$$

subject to:

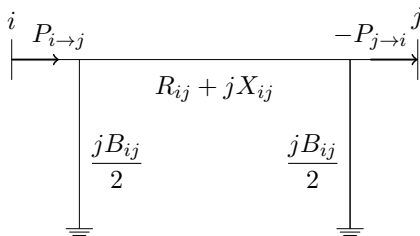
$$P_{G_i}^{min} \leq P_{G_i} \leq P_{G_i}^{max}$$

$$\mathbf{B} \cdot \theta = \mathbf{P}_G - \mathbf{P}_D$$

$$P_{ij,max} \leq \frac{1}{x_{ij}} (\theta_i - \theta_j) \leq P_{ij,max}$$

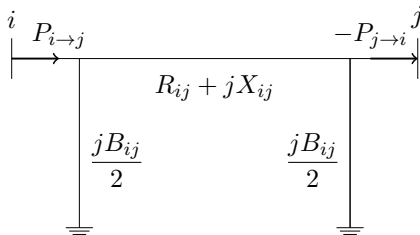
- A DC-OPF with quadratic costs is a convex problem
- $\frac{1}{2}x^T Qx + c^T x$: How does Q look like in a 'QP' DC-OPF?

Active Power Losses in AC-OPF



π -model of the line

Active Power Losses in AC-OPF



π -model of the line

- Losses = “P leaving node i ” – “P arriving at node j ”
- P leaving node i : $P_{i \rightarrow j}$
- P arriving at node j : $-P_{j \rightarrow i}$

$$P_{\text{losses}} = P_{i \rightarrow j} + P_{j \rightarrow i}$$