Stochastic AC Optimal Power Flow with Approximate Chance-Constraints

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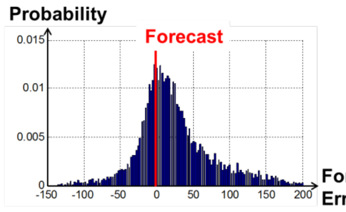
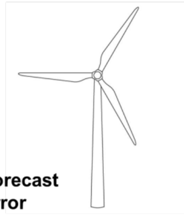
Uncertain power production ➡ Uncertain power flows

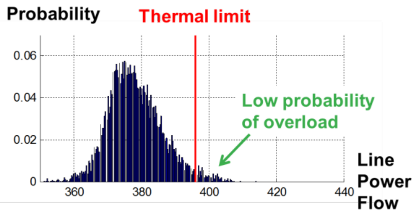




Uncertainty from:

- Renewables
- Intra-day trading
- ...

GOAL: Secure system operation despite uncertainty



Approach:

AC optimal power flow with **approximate chance constraints**

1. Chance-Constrained AC Optimal Power Flow

- Formulation is based on full AC power flow equations
- Power injections $\tilde{P}, \tilde{Q}$ are uncertain
  → Voltages $\tilde{\Theta}, \tilde{V}$ and currents $\tilde{I}$ are uncertain!
- Chance constraints limit the probability of overload

$$\min_{P_G} \quad \sum_{i\in\mathcal{G}}\left(c_{2,i}P_{G,i}^2 + c_{1,i}\,P_{G,i} + c_{0,i}\right)$$

$$\begin{aligned}\text{s.t.}\quad & P_G^{min} \le P_G \le P_G^{max} \\ & Q_G^{min} \le Q_G \le Q_G^{max} \\ & f\left(\tilde{\Theta}, \tilde{V}, \tilde{P}, \tilde{Q}\right) = 0\end{aligned}$$

AC power flow equations

$$\begin{aligned}&\mathbb{P}\left(\tilde{I}_{L,j} \le I_{L,j}^{max}\right) \ge 1-\epsilon, \quad j \in \mathcal{L} \\ &\mathbb{P}\left(\tilde{V}_i \le V_i^{max}\right) \ge 1-\epsilon, \quad i \in \mathcal{B} \\ &\mathbb{P}\left(V_i \ge V_i^{min}\right) \ge 1-\epsilon, \quad i \in \mathcal{B}\end{aligned}$$

Chance constraints on currents and voltages

2. Approximate Chance-Constraint Reformulation

Step A: Linearize around expected operating point

- Formulate power flow for expected P, Q: $f(\Theta, V, P, Q) = 0$
- Linearize with respect to $\Delta P, \Delta Q$: $\tilde{I}_{L,j} \approx I_{L,j} + \Gamma_{I\,(.,j)}\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}$

Full AC

Γ_I **Linear sensitivity factor**

- Approximate chance-constraint: $\mathbb{P}\left(I_{L,j} + \Gamma_{I(.,j)}\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} \le I_{L,j}^{max}\right) \ge 1-\epsilon$

Step B: Analytic reformulation based on Gaussian uncertainty

- Assume Gaussian forecast errors $\Delta P, \Delta Q$
- Analytical reformulation:

$$I_{L,j} \le I_{L,j}^{max} - \boldsymbol{\Phi}^{-1}(1-\epsilon)\sqrt{\Gamma_{I(.,j)}\boldsymbol{\Sigma_W}\,\Gamma_{I(.,j)}}$$

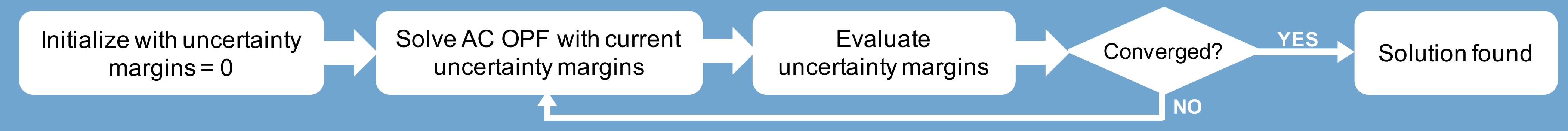
Nominal AC solution

«Uncertainty margin» Ω_V based on linearization

$\boldsymbol{\Sigma_W}$ **Covariance matrix**

$\boldsymbol{\Phi}$ **Gaussian cumulative distribution function**

3. Iterative solution algorithm



Initialize with uncertainty margins = 0

Solve AC OPF with current uncertainty margins

Evaluate uncertainty margins

Converged?

YES

Solution found

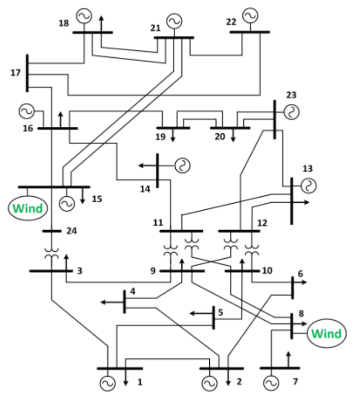
NO

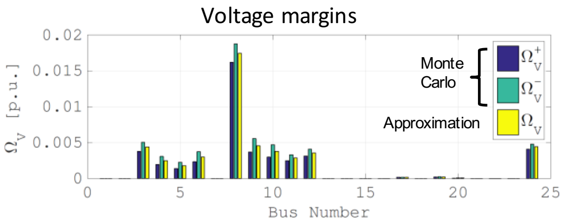
4. Case study

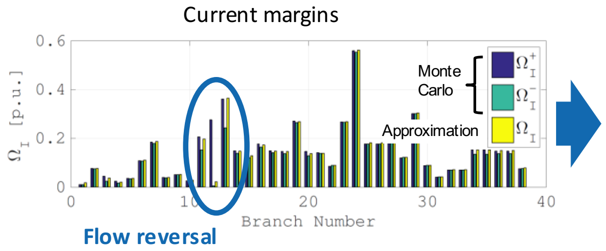
- IEEE RTS 96 with two wind power plants
- Violation probability $\epsilon = 0.05$

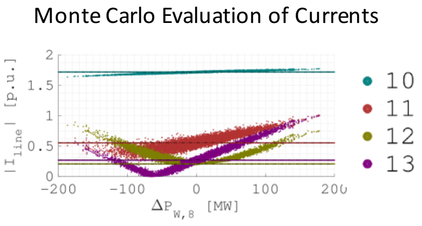
Convergence and accuracy

- **Fast convergence** of uncertainty margins
- Reasonably **accurate approximation**, except for lines with flow reversal



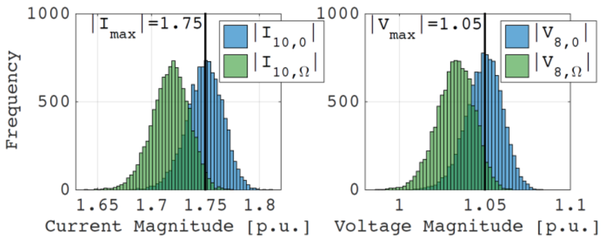






Performance of the AC OPF Solution

Violation probability decreases from 50% to below 5% with chance-constraints



Conclusions

- Computationally tractable and reasonably accurate way to account for uncertainty in AC optimal power flow
- Can be implemented within existing AC OPF tools
- Can be extended to other probabilistic constraints and non-normal uncertainty distributions

Poster presenter: Line Roald

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