Security Constrained OPF Incorporating Corrective Control of HVDC

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***Abstract*—This paper introduces linear current distribution factors for use in: (a) SCOPF formulations, and (b) in analytical approximations of corrective control actions for HVDC lines. These are relationships based on complex numbers, which can be used in the context of full AC power flow equations. Three distribution factors are introduced: the linear AC outage distribution factor (LOCDF), the current distribution factor (CDF), and a linear factor which expresses the effect of HVDC corrective control actions on the line currents. Through case studies we show that the accuracy of the proposed linear factors is comparable to existing, more complex, methods. We further demonstrate their applicability in an SCOPF problem, incorporating the corrective control capabilities of HVDC. The use of the proposed linear factors results in faster computation times, as they can be precomputed before the execution of the SCOPF algorithm. Furthermore, taking advantage of their linear properties, we introduce an approximate analytical solution for corrective control of HVDC lines. We apply this closed form relationship on a case study for line outages.**

***Index Terms*—High Voltage Direct Current, Voltage-Source Converter, corrective control, security-constrained optimal power flow, distribution factors, analytical solution, current injection method**

I. Introduction

In order to ensure security, power system operation should always fulfill certain security criteria. Among them, most commonly used is the N-1 criterion. An optimization problem taking explicitly into account the N-1 criterion is often termed Security-Constrained Optimal Power Flow (or SCOPF).

In its general formulation, the SCOPF is a non-linear, non-convex, large-scale optimization problem with both continuous and discrete variables [1]. One of the major SCOPF challenges is the large problem size. For a complete survey of several SCOPF formulations and the main challenges the reader can refer to [2]. In this paper, we propose linear factors within an AC power flow context, capable of computing the line currents after a contingency. The goal is to decrease complexity and computation time.

In its first formulations, e.g., [1], the SCOPF focussed on preventive control actions. The first SCOPF formulation incorporating corrective control actions from generation rescheduling and switching actions was presented in [3]. With the incorporation of modern fast power flow control elements in power systems, such as VSC-HVDC lines, the need for identifying corrective control actions becomes even more apparent. The incorporation of the corrective control capabilities of VSC-HVDC lines in an SCOPF algorithm was first presented in [4]. In this paper we propose a more approximate method based on linear factors, that leads to faster computation time.

Three distribution factors are derived: the linear AC outage distribution factor (LOCDF), which determines the line current flows after the occurrence of a line outage; the current distribution factor (CDF), which computes the line currents after a generation outage; and a linear factor which expresses the effect of HVDC corrective control actions on the line currents ($\text{LOCDF}^{\text{PCC}}$). The derivation of LOCDF is based on the Current Injection method, first proposed in [5], [6]. The basic current injection method requires the solution of a linear system for the calculation of the line flows after a line outage. Extensions of this method for increased accuracy have already been proposed [7]. This paper focusses on the basic current injection method and extends it in order to avoid the need for a linear system, and instead use the proposed linear factors. The CDF has already been introduced in [5], [6] under a different name; here we extend its use to account for generation outages. The $\text{LOCDF}^{\text{PCC}}$ is a function of LOCDF and CDF. The use of the proposed linear factors results in faster computation times, as they can be precomputed before the execution of the SCOPF algorithm.

Through the derivation of these factors, we achieve two goals. First, we extend the range of application of the current injection method by introducing a faster and simpler approach. These factors can be precomputed, similar to the distribution factors in a DC-OPF formulation. Second, these linear relationships allow the incorporation of the HVDC corrective control capabilities in the SCOPF problem.

This paper is organized as follows. Section II presents the Current Injection method. Section III derives the linear factors and examines their accuracy through case studies. In Section IV we describe how these factors can be incorporated in an SCOPF problem, while in Section V we demonstrate the capabilities of such an algorithm on a case study. In Section VI, we further introduce an approximate analytical solution for corrective control of VSC-HVDC lines. This closed-form relationship can be used as a first step for a fast computation of an HVDC corrective control action. Finally, Section VII concludes this paper.

II. Current Injection Method

The current injection method is able to determine accurately the line currents after an outage through the solution of a linear system of equations, thus avoiding the need of a full power flow calculation. As a result, constraints for the line currents after an outage can be easily incorporated in the

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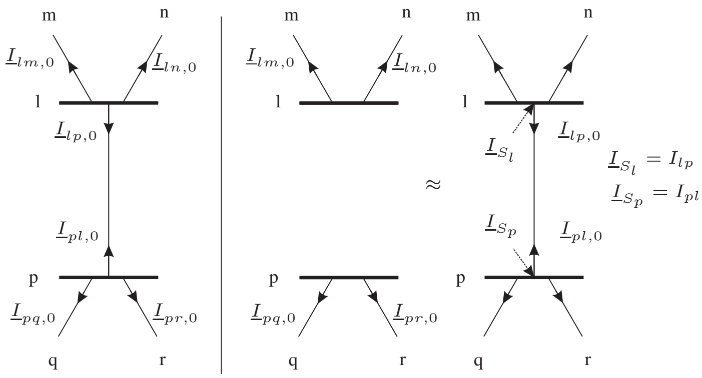

Fig. 1: Illustration of the Current-Injection Method.

optimization problem. Figure 1 illustrates the concept of the current injection method. The left part shows the system state before the outage, while in the middle the situation after the outage is represented. As shown in the right part of Fig. 1, the outage of the line $l - p$ can be represented by a pair of injection currents $\underline{I}_{S_l}, \underline{I}_{S_p}$ at the respective nodes, approximately compensating for the current flowing over the line. Essentially, instead of changing the network topology, injection currents eliminate the line flow, thus simulating the line outage.

Following the notation in [7], in the non-faulted situation the relationship between bus voltages and bus currents is given by (1). $\underline{Z}_0$ is the bus impedance matrix. When a line outage occurs, bus voltages and bus currents change to $\underline{V}_F$ and $\underline{I}_F$ respectively, while the new network topology leads to a new bus impedance matrix $\underline{Z}_F$. Equation (2) holds in this case.

$$\underline{V}_0 = \underline{Z}_0 \cdot \underline{I}_0 \tag{1}$$

$$\underline{V}_F = \underline{Z}_F \cdot \underline{I}_F \tag{2}$$

Here, we introduce the vector of injection currents $\underline{I}_S$. Assuming a non-faulted network topology, as given by $\underline{Z}_0$, the injection currents should compensate for the flows on the outaged line. The only non-zero elements of the vector $\underline{I}_S$ correspond to the nodes l, p, where the outage occurred. Hence, (3) is equivalent to (2):

$$\underline{V}_F = \underline{Z}_0 \cdot (\underline{I}_F + \underline{I}_S) \tag{3}$$

As a result, the change in the bus voltages after the fault is given by:

$$\Delta\underline{V} = \underline{V}_0 - \underline{V}_F = \underline{Z}_0 \cdot (\underbrace{\underline{I}_0 - \underline{I}_F}_{\approx 0} - \underline{I}_S) \tag{4}$$

As implied by (4), the basic current injection method assumes that the difference in the bus currents before and after the outage is negligible. Hence, the changes in the line flows can be computed through (5):

$$\Delta\underline{I}_{line} = \underline{I}_{line,0} - \underline{I}_{line,F} = \underline{Y}_L \Delta\underline{V} \approx -\underbrace{\underline{Y}_L \underline{Z}_0}_{\underline{D}} \cdot \underline{I}_S \tag{5}$$

Here, $\underline{Y}_L$ is the line admittance matrix of the situation without outage. Matrix $\underline{D} = \underline{Y}_L \cdot \underline{Z}_0$ is known as the matrix of distribution factors [5], reflecting the influence of the injection currents to the line currents.

The injection currents that we calculate have an effect not only on the rest of the line flows but also on the outaged line itself, in a recursive way. In other words, any additional current injection at the nodes l, p will also change the line flow to $\underline{I}_{lp,new} = \underline{I}_{lp,0} - \Delta\underline{I}_{lp}$. As a result, if we wish to eliminate the line flows on the outaged line, the injection currents should be determined from the following equations:

$$\underline{I}_{S_l} = \underline{I}_{lp,0} - \Delta\underline{I}_{lp} = \underline{I}_{lp,0} + \underline{D}_{(lp,l)} \cdot \underline{I}_{S_l} + \underline{D}_{(lp,p)} \cdot \underline{I}_{S_p} \tag{6}$$

$$\underline{I}_{S_p} = \underline{I}_{pl,0} - \Delta\underline{I}_{pl} = \underline{I}_{pl,0} + \underline{D}_{(pl,l)} \cdot \underline{I}_{S_l} + \underline{D}_{(pl,p)} \cdot \underline{I}_{S_p} \tag{7}$$

where $\Delta\underline{I}_{lp}$ corresponds to the element associated with the line $l - p$ in the vector $\Delta\underline{I}_{line}$, while $\underline{I}_{lp,0}$ is the current that flows on the line before the outage. The notation $\underline{D}_{lp,l}$ indicates the element in the row associated with line $l - p$ and in column p of matrix $\underline{D}$. Using the values for $\underline{I}_{S_l}$ and $\underline{I}_{S_p}$ calculated through the above equations, the changes in voltages and line currents after the outage can consequently be calculated with the help of (4) and (5).

III. Current Distribution Factors

A. AC Current Distribution Factor

In Section II we introduced matrix $\underline{D}$, which is the matrix of distribution factors reflecting how bus current injections change line currents. We define matrix $\underline{D} = \underline{Y}_L \underline{Z}_0$ as the product of the line admittance matrix $\underline{Y}_L$ and the bus impedance matrix $\underline{Z}_0 = \underline{Y}_0^{-1}$ (This matrix was first introduced in [5]).

Besides its use in the current injection method, matrix $\underline{D}$ can generally determine the relationship between the bus current injections $\underline{I}_0$ and the line currents $\underline{I}_{line}$, as shown in (8):

$$\underline{I}_{line} = \underline{Y}_L \cdot \underline{V}_0 = \underbrace{\underline{Y}_L \cdot \underline{Y}_0^{-1}}_{=\underline{D}} \underline{I}_0 \tag{8}$$

For the rest of this paper we will refer to matrix $\underline{\mathrm{D}}$ as the matrix of Current Distribution Factors (CDF) and define it as in (9):

$$\text{CDF:} \qquad \underline{D} = \underline{Y}_L \underline{Y}_0^{-1} \tag{9}$$

B. Linear Outage Current Distribution Factors (LOCDF)

As described in Section II, the outage of the line $l - p$ can be represented by a pair of injection currents $\underline{I}_{S_l}, \underline{I}_{S_p}$ at the respective nodes, approximately compensating for the current flowing over the line. Using the values for the injection currents $\underline{I}_{S_l}$ and $\underline{I}_{S_p}$ calculated through (6) and (7), the changes in voltages and line currents after the outage can consequently be computed through these injection currents and matrix $\underline{D}$ (see (4) and (5)), i.e.:

$$\Delta\underline{I}_{line} = -\underline{D} \cdot \begin{bmatrix} 0 \\ \underline{I}_{S_l} \\ \vdots \\ \underline{I}_{S_p} \\ 0 \end{bmatrix} \tag{10}$$

Examining $\underline{I}_{S_l}$ and $\underline{I}_{S_p}$ in simulations, we observe that their *absolute* values are quite similar. This is also anticipated from circuit theory. The bus injection currents try to eliminate the flow on the outaged line by injecting opposite currents at the line ends. The value of the current at the beginning of the line differs from the value of the current at the end of the line only by the amount of the leakage current, which is represented though the shunt impedances in the line pi-model. Considering that the losses corresponding to such currents are a very small percent of the line flow, it follows that currents $\underline{I}_{S_l}$ and $\underline{I}_{S_p}$ would also be very similar. By assuming that $\underline{I}_{S_l} = -\underline{I}_{S_p}$ allows us to derive a linear factor to describe the effect of one line outage on all line flows, which is dependent only on the electrical characteristics of the network, i.e. dependent only on matrix $\underline{D}$ and not on the operating point. These factors can be pre-computed and kept constant during the whole optimization procedure. As we will see in the following sections, the accuracy of this factor is equally good as the basic current injection method presented in [5] and [6].

From (6) and (7), by assuming that $\underline{I}_{S_l} = -\underline{I}_{S_p}$, we have:

$$\underline{I}_{S_l} = \frac{1}{1 - \underline{D}_{(lp,l)} + \underline{D}_{(lp,p)}} \cdot \underline{I}_{lp,0} \tag{11}$$

$$\underline{I}_{S_p} = \frac{1}{1 + \underline{D}_{(pl,l)} - \underline{D}_{(pl,p)}} \cdot \underline{I}_{pl,0} \tag{12}$$

In the general case:

$$\Delta \underline{I}_{line} = -\underline{D} \cdot \underline{S} \cdot \underline{I}_{line} \tag{13}$$

where $\underline{S}$ is a $n_B \times 2n_L$ matrix. Matrix $\underline{S}$ has $2n_L$ columns since we take into account both flow directions in our calculations (n_B and n_L are the number of buses and lines respectively). When considering line $l-p$ as the only critical outage, then matrix $\underline{S}$ will be a matrix having non-zero elements only in the vectors corresponding to the outaged line $l-p$, and its reverse flow $p-l$, i.e.:

$$\underline{S}_{lp} = \begin{bmatrix} 0 \\ \frac{1}{1+\underline{D}_{(lp,l)}-\underline{D}_{(lp,p)}} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad \underline{S}_{pl} = \begin{bmatrix} 0 \\ \vdots \\ \frac{1}{1-\underline{D}_{(pl,l)}+\underline{D}_{(pl,p)}} \\ \vdots \\ 0 \end{bmatrix} \tag{14}$$

$$\underline{S} = \begin{bmatrix} \underline{0} & \underline{S}_{lp} & \dots & \underline{0} & \underline{S}_{pl} & \dots & \underline{0} \end{bmatrix} \tag{15}$$

Specifically, the non-zero element of $\underline{S}_{lp}$ will appear in the row corresponding to bus l, and the non-zero element of $\underline{S}_{pl}$ will appear in the row corresponding to bus p.

The line currents after an outage will be given from the relationship:

$$\underline{I}'_{line} = (\underline{1} + \underline{D} \cdot \underline{S}) \cdot \underline{I}^{init}_{line} \tag{16}$$

Equation (17) gives the definition of the Linear Outage Current Distribution Factor (LOCDF):

$$LOCDF = \underline{1} + \underline{D} \cdot \underline{S} \tag{17}$$

C. Examining the LOCDF accuracy: 10-bus system

We test the accuracy of our approach on the 10-bus system presented in Section V. First, we will compare the post-contingency current flows computed through the current distribution factors and the basic current injection method. Subsequently, we will focus on the LOCDF approximation errors with respect to the flows resulting from an AC Power Flow.

Figures 2 and 3 present the deviation of the LOCDF calculation for the post-contingency currents with respect to the basic current injection method for all possible contingencies, i.e. line outages. The deviation has been calculated here in two ways. First, by computing the deviation of the current magnitudes only, without taking into account the phase angle difference, as shown in (18). The resulting deviations are presented in Fig. 2.

$$\text{Magn.Dev.} = \frac{|I_{\text{Curr.Inj}}| - |I_{\text{LOCDF}}|}{|I_{\text{Curr.Inj}}|} \tag{18}$$

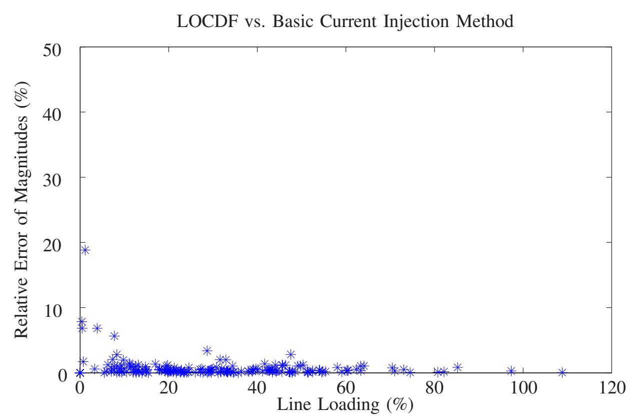

Fig. 2: LOCDF vs. Basic Current Injection: Deviation of current *magnitude* plotted against the corresponding line loading for the 10-bus system.

Second, by calculating the error vector magnitude (EVM) as shown in (19), which takes both magnitude and phase angle differences into account. Figure 3 presents the EVM deviations.

$$\text{EVM} = \frac{|I_{\text{Curr.Inj}} - I_{\text{LOCDF}}|}{|I_{\text{Curr.Inj}}|} \tag{19}$$

For estimating the line loading, only the current magnitude is of interest. As shown in Fig. 2, the difference between the LOCDF calculation and the basic current injection method is very small. For line loadings above 50% of the line thermal capacity, the relative deviation does not exceed 2% in any case. In lower loadings, the error calculation is more sensitive to small deviations and therefore we observe higher relative deviations. For example, the post-contingency current flow of a line loaded at 1.12% of the thermal capacity differs by about 38% between the two methods. Nevertheless, in absolute numbers the line is loaded at 1.62% of its thermal capacity by the LOCDF method, while by the basic current injection method the line loading is estimated at 1.18% of its thermal

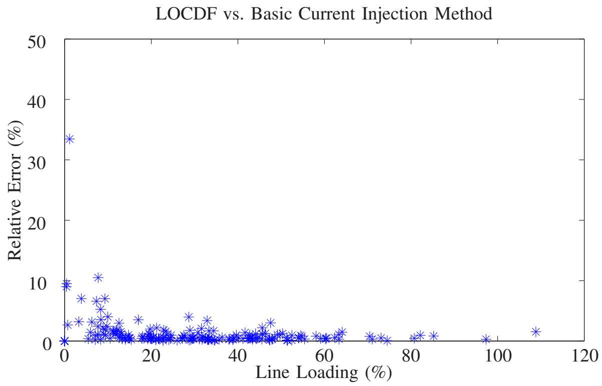

Fig. 3: "Error Vector": Deviation of LOCDF calculation in % from the basic current injection method plotted against the corresponding line loading for the 10-bus system.

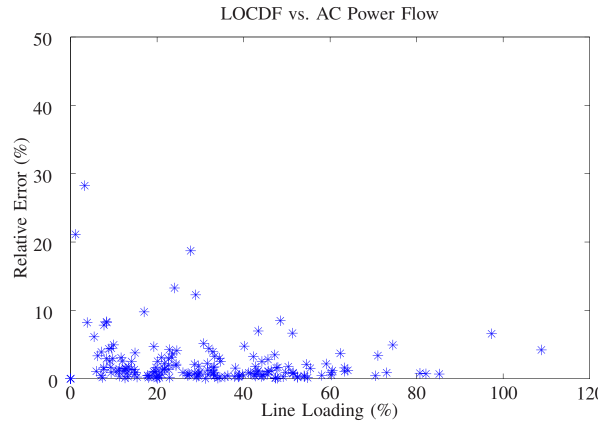

Fig. 4: Approximation errors of LOCDF calculation (EVM) plotted against the corresponding line loading for the 10-bus system.

capacity. As it can be observed the difference with respect to the absolute line loading is almost insignificant.

As we wish to use the CDF approximations for general calculations, e.g. also for estimating active and reactive power and not only line overloadings, the phase angle difference plays also a role. The error vector magnitude also includes such deviations. By definition, the magnitude deviations (see (18)) will always be smaller than the EVM deviations (see (19)) as they do not include the difference in phase angles. This is also evident by comparing Fig. 3 with Fig. 2. We see, nevertheless, that the difference between the two methods is small. For line loadings above 50%, the difference does not exceed 3% in any case.

Concluding, we can see that LOCDF calculation yields almost the same results as the basic current injection method, especially with increasing line loadings. For very low line loadings, the relative deviation increases, but remains small in absolute values.

In the following investigations in this section we will examine only the error vector magnitude, as this includes the phase angle difference and poses an upper bound on the current magnitude error. The approximation error for the current magnitude, which plays an important role for the calculation of post-contingency overloadings, will always be smaller than the error vector magnitude. We leave the comparison between the two approximation methods, and we now focus on the actual error that the LOCDF calcuation introduces in the computation of the post-contingency current flow. Figure 4 presents the relative errors of the post-contingency line flows computed with the LOCDF, as opposed to the actual values of line currents, obtained through an AC Power Flow. The relative errors are plotted against the corresponding loadings of the lines. As it can be observed, for higher line loadings the accuracy of the LOCDF factor increases. For loadings above 50%, the approximation error is below 13%. Similar results are obtained by the basic current injection method when compared with the results from an AC Power Flow [7].

As it can be observed, the LOCDF calculation results to acceptable approximation errors, comparable to the basic current injection method. It is also important to note that not all of these errors are critical for the power system operation, especially if one focuses on post-contingency line overloadings. In the following paragraph we will examine how these approximation errors can lead to a false estimate for a line overloading in the IEEE 118-bus system. We will find out that the percentage of false-negative or false-positive samples is indeed very small. Taking further into account the gains in computation speed, as we will see in Section V, the LOCDF provides a good and fast estimate of the post-contingency current flows in a security-constrained AC-OPF context.

D. Examining the LOCDF accuracy: IEEE 118-bus System

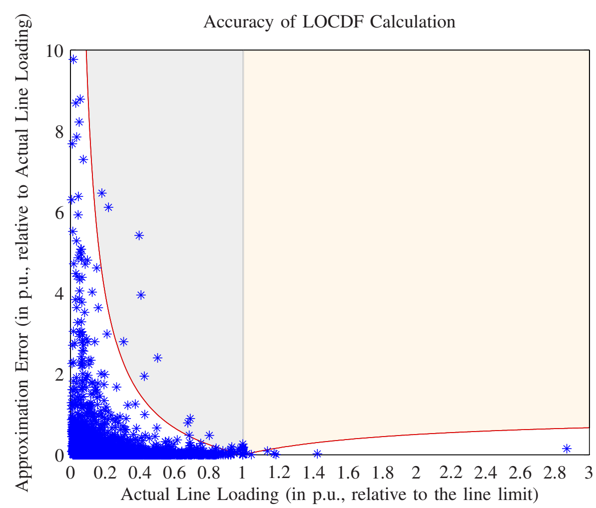

Fig. 5: Approximation errors of LOCDF calculation plotted against the corresponding line loading. The grey area denotes the false positive samples and the light orange the false negative.

The LOCDF accuracy was also tested in the IEEE 118-bus system. The system data can be found in [8], while the line capacity data were taken from [9]. The system consists of 186 lines. From these, the outage of 9 lines results to isolated buses, and therefore they should be considered as a bus outage and not a line outage. As a result, we studied 177 line outages and their effect on the line flows (total 32922 samples). Fig. 5 presents the approximation errors of the LOCDF calculation against the corresponding line loading. In this paragraph, we focus on the number of false-positive and false-negative samples. False-positive are loadings that were assumed to exceed the line limit, but in reality were not. In an SCOPF context, these samples will result in constraint violations, for which the SCOPF algorithm has to account. False-negative are loadings that were considered to be below the line limit, but in reality the lines were overloaded. False-negative samples do not appear to violate the SCOPF constraints, although in reality an SCOPF algorithm should have accounted for these instances too. In total, only 27 out of 32922 (0.08%) samples were false positive and 1 sample (0.00%) false negative.

Concluding, we can see that for the post-contingency flows, which are of importance in a security-constrained OPF algorithm, only in 0.08% of the cases were found to effectively influence the dispatch determined through the optimization algorithm. As an additional comment here we could mention that considering that the number of false-positive samples is higher than the false-negatives, in this case (IEEE 118-bus) the SCOPF algorithm will probably tend to result to a more constrained, i.e. more expensive, dispatch than it would in reality.

IV. Security-Constrained OPF based on Current Distribution Factors

The SCOPF algorithm extends the AC-OPF problem by including additional constraints for the line flows in case of component outages. In this section we will outline the formulation of these constraints in case of AC line, generation, and HVDC outages, based on the linear relationships introduced in the previous sections. The rest of the constraints, which correspond to an AC-OPF problem, including the HVDC lines, are the same as described in Ref. [4]. It should be noted that exhaustive SCOPF formulations also include corrective actions from switching operations, generators, and other components. Different strategies result in different priorities concerning which corrective actions should be implemented first after a contingency. This is outside the scope of this paper, as the focus here is to demonstrate how the introduced linear relationships perform with respect to accuracy and computation costs in an SCOPF context.

A. Inclusion of Line Outages and Generation Outages

AC Line Outages: In case of AC line outages, as already presented in Section III-B, the LOCDF can determine with a good approximation the post-contingency current flow on all lines.

Generation Outages: In case of a generation outage, the bus current vector $\underline{I}_0$ will change. The lost generated power should be "shifted" to other generators. Since the generating buses are PV buses, by assuming constant voltage before and after the outage on all generating buses, the power shifting is equivalent to the shifting of current injections. Let a generator on bus p get outaged, and generators on buses m and n compensate for this generation loss (N.B. on buses m, n, p loads may also exist). Then:

$$\underline{I}'_{bus} = \begin{bmatrix} \vdots \\ I_{0_p} - I_{g_p} \\ I_{0_m} + \Delta I_{g_m} \\ I_{0_n} + \Delta I_{g_n} \\ \vdots \end{bmatrix} \quad \text{with} \quad I_{g_p} = \Delta I_{g_m} + \Delta I_{g_n} \tag{20}$$

Through (20) and (8) we observe that in case of a generator outage the current distribution factors, represented by the matrix $\underline{D}$, can determine the line flows:

$$\underline{I}'_{line} = \underline{D} \cdot \underline{I}'_{bus} \tag{21}$$

HVDC Outages: We can handle HVDC outages similar to generation outages, since HVDC converters are modelled as virtual voltage sources [10]. Instead of generation shifts, we would need to set to zero the entries in $\underline{I}'_{bus}$ that correspond to the HVDC converters. The new line currents are expressed again through (21).

B. VSC-HVDC Corrective Control through Current Distribution Factors

The Current Distribution Factors (CDF) and the Linear Outage Current Distribution Factors (LOCDF) can be used for the calculation of the post-contingency control actions of a VSC-HVDC line. A contingency could be either a generator or a line outage. Here, we will focus on the more interesting case of a line outage. A VSC-HVDC line can be modelled as two voltage sources, with one voltage source connected at each end of the line. One of the two voltage sources will be withdrawing a certain amount of power, while the voltage source at the other end will be injecting the same amount of power. Here, we neglect the ohmic losses and the converter losses of the HVDC line. We also assume that the HVDC line operates in PV mode, setting the voltage at both line ends to a specified value. This allows us dealing with the currents instead of active power for the post-contingency control, without significant loss of accuracy. Let the HVDC line be connected between nodes $m - n$. Then a change ΔI_{DC} in the HVDC line flow will change the bus injection currents:

$$\underline{I}'_{bus} = \begin{bmatrix} I_{0_1} \\ \vdots \\ I_{0_m} \\ I_{0_n} \\ \vdots \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ \Delta I_{DC} \\ -\Delta I_{DC} \\ \vdots \end{bmatrix}, \tag{22}$$

which will result in a change of the line currents, as expressed through (21).

Assume now that we want to study the change on AC line flow k, when AC line $l - p$ is outaged, and the HVDC line $m - n$ changes its power flow. Then the current flow on line k will be given from the relationship:

$$I_k^{lp_{out}} = LOCDF_{k,lp} \cdot \underline{I}'_{line,\Delta I_{DC}}, \tag{23}$$

where $\underline{I}'_{line,\Delta I_{DC}}$ are the changes in the line currents solely due to the change in the HVDC power flow.

From (21) we rewrite (23) as follows:

$$I_k^{lp_{out}} = LOCDF_{k,lp} \cdot \underline{D} \cdot \underline{I}'_{bus,\Delta I_{DC}}. \quad (24)$$

$LOCDF_{k,lp}$ is given by:

$$LOCDF_{k,lp} = 1 + \underline{D}_k \cdot \underline{S}^{lp_{out}}, \quad (25)$$

where $\underline{D}_k$ is the row k of the matrix $\underline{D}$ and

$$\underline{S}^{lp_{out}} = \begin{bmatrix} \underline{0} & \underline{S}_{lp} & \dots & \underline{0} & \underline{S}_{pl} & \dots & \underline{0} \end{bmatrix}. \quad (26)$$

So, in general:

$$\underline{I}'_{line} = LOCDF \cdot \underline{D} \cdot \underline{I}'_{bus}. \quad (27)$$

Expressing the line currents as a function of the HVDC corrective control, we arrive at the following equation:

$$\underline{I}'_{line} = LOCDF \cdot \underline{D} \cdot \underline{I}_0 + LOCDF \cdot \underline{D} \cdot \underline{I}_{DC}, \quad (28)$$

where $\underline{I}_0$ is the vector of the pre-fault bus currents and $\underline{I}_{DC}$ is a vector as shown in (22).

V. Case Study

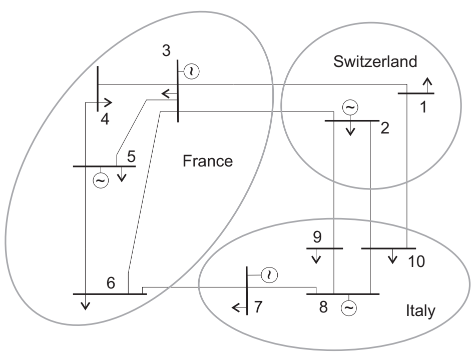

Fig. 6: 10-bus network used for the case studies.

In this section we present a case study applying the Security-Constrained OPF based on the current distribution factors on a 10-bus system. The system, depicted in Fig. 6, emulates to a certain extent the flows between Switzerland, France, and Italy. The system data and description can be found in [11]. For the sake of comparison with the SCOPF based on the more accurate Current Injection method, we will use the same assumptions as in Case Study #2 of Ref. [4]. As critical contingency we also assumed here the outage of line 6-7, while an HVDC line is replacing the AC line between nodes 1-10. This SCOPF formulation is again based on the full AC equations. In this formulation, instead of solving a linear system as in Ref. [4], we use (27) to consider the line outages and the corrective control actions of the HVDC. This reduces significantly the computation effort, since the matrices $LOCDF$ and $\underline{D}$ can be precomputed. Both SCOPF formulations have been solved with the use of SNOPT solver from Tomlab Optimization (www.tomopt.com).

Table I compares the generation costs obtained from both algorithms. The formulation based on the current injection method results in higher costs, which are closer to reality, as the approximations introduced with the current distribution factors relax the constraints in this case. Still, the deviation as can be observed, is relatively small. Especially in the case where corrective control actions are enabled, the difference between the two methods is almost insignificant. The reason lies probably in the fact that the additional degrees of freedom after an outage relax the security constraints. Thus, the solution is mainly constrained from the AC-OPF equations (see for example Eq. 2 – Eq. 16 in [4]). As both problems share the same AC-OPF formulation, the solutions converge.

TABLE I: Total Generation Costs. Comparison between the method based on the Current Distribution Factors and the method based on Current Injection [4] (Critical Contingency: outage of line 6-7).

	SCOPF without Corr.Ctrl.	SCOPF with Corr.Ctrl
Current Dist. Factors	321'926 €/h	315'443 €/h
Current Injection	324'816 €/h	316'328 €/h
Deviation	0.9%	0.2%

TABLE II: HVDC active power setpoints during the pre-fault and post-fault state resulting from the CDF-based SCOPF and the Current-Injection-based SCOPF (P_{HVDC} positive direction: $1 \rightarrow 10$).

	SCOPF (pre-fault dispatch) [MW]	SCOPF (HVDC Corr.Ctrl.) [MW]
CDF : P_{HVDC}	-568.8	1215.6
Curr. Inj.: P_{HVDC}	-192.3	1400.1

Table II presents the setpoints for the HVDC line calculated with both methods. We observe here that there is a difference between the solutions. For the pre-fault dispatch, although the two SCOPF formulations converge to different HVDC setpoints, both HVDC power flows result to no overloaded lines. Concerning the post-fault state, beside Table II in Fig. 7 we examine in more detail the line loadings resulting from the post-contingency HVDC setpoints. We plot the line loadings as computed through the CDF and the Current Injection method, and compare them with the results from an AC Power Flow, assuming the same HVDC setpoints as in the respective formulation. As we can see in Fig. 7, line 2-10 results in a higher actual overloading in the CDF-based SCOPF (AC Power Flow – CDF bar). This occurs due to the higher level of approximation of the Current Distribution Factors.

In Table III we compare the computation effort for both SCOPF formulations. As we observe, the formulation based on the CDF requires about 50% of the time required for the solution based on the current injection method for this small system. It is expected that in larger systems the benefits in terms of computation effort will be higher.

TABLE III: Computation time needed for the CDF-based SCOPF and for the Current-Injection-based SCOPF.

	CDF	Curr. Injection
Comp. Time	0.62 seconds	1.16 seconds

Before concluding this paper, in the next section we employ the Current Distribution Factors to derive an approximate

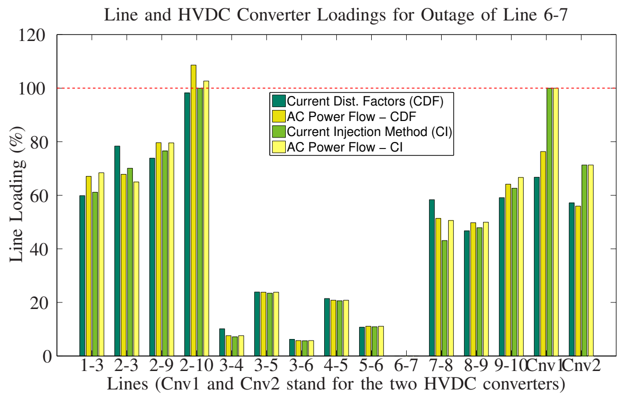

Fig. 7: Comparison of the line loadings computed through the CDF-based SCOPF and the Current-Injection-based SCOPF. The yellow bars (AC Power Flow) present the actual loadings appearing in the system with the Generation+HVDC dispatch as determined from the two SCOPF formulations.

analytical solution for VSC-HVDC corrective control. This allows us to determine an HVDC power setpoint in order to relieve a line overloading, without the need to run an SCOPF.

VI. Approximate analytical solution for VSC-HVDC corrective control

The formulations presented in Section IV allow us to introduce an approximate analytical solution for the post-contingency control actions of the VSC-HVDC lines. This solution could be of help during the real-time operation of power systems. Assuming that an outage leading to a line overloading occurs, the operator could use this method to compute fast an HVDC corrective control action which could help relieve the overloaded line.

Assume that an outage of line $l-p$ leads to an overloading of line k, equal to $\underline{I}_k^{lp_{out}}$. In order to achieve a line current $\underline{I}_k'$ which will lie below the thermal limit, we assume a uniform decrease of the absolute values of both the real and imaginary part of $\underline{I}_k^{lp_{out}}$ so that $|\underline{I}_k'| = F_k$. This is shown in (29). F_k is a scalar and represents the current thermal limit of the line.

$$\underline{I}_k' = F_k \cdot \frac{\underline{I}_k^{lp_{out}}}{|\underline{I}_k^{lp_{out}}|} \tag{29}$$

In our analysis we are dealing with complex currents. We employ (29), in order to determine a line current which will have both a real and imaginary part, but its absolute value will be equal to the thermal limit. Here, we assume that this current will have a phase angle similar to $\underline{I}_k^{lp_{out}}$, as the electrical characteristics of the network and the bus current injections, except for the HVDC flow, remain constant.

From (27), we define $LOCDF^{PCC} = LOCDF \cdot \underline{D}$. PCC stands for "post-contingency control", which is a term equivalent to "corrective control". Assuming that we want to control a single HVDC line which is connected between the nodes C_m and C_n, then the change in the HVDC line flow should be:

$$\Delta I_{C_m} = \frac{\underline{I}_k' - LOCDF_k^{PCC}\underline{I}_{bus}}{LOCDF_{k,C_m}^{PCC} - LOCDF_{k,C_n}^{PCC}} \tag{30}$$

Here, $\underline{I}_{bus}$ is the vector with the bus currents before the outage. $LOCDF_k^{PCC}$ refers to the row k of the matrix, while $LOCDF_{k,C_m}^{PCC}$ refers to the matrix element.

From the change in the current ΔI_{C_m} we can subsequently calculate the new active power flow setpoint of the HVDC line.

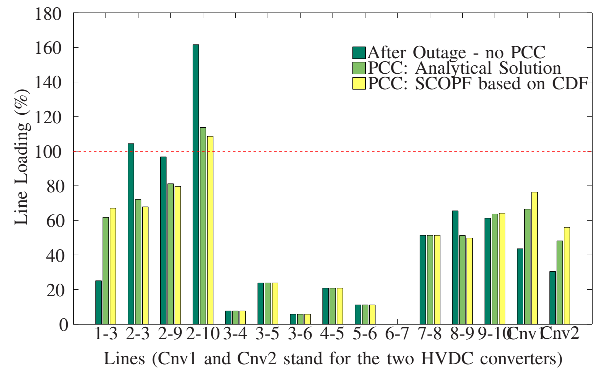

Fig. 8: Comparison of the line loadings after the outage of line 6-7: (a) without any post-contingency control actions (PCC), (b) PCC through the analytical approach, (c) (PCC) through the SCOPF based on the Current Distribution Factors.

In Fig. 8, we present the resulting line loadings after applying (30) in the case study we presented in Section V. The dark green bars present the line loadings exactly after the line outage 6-7, when no post-contingency control action (i.e. corrective control) is taken. The light green bars present the line loadings that occur if we apply the post-contingency control action computed through the analytical formulation in (30). The yellow bars show the line loadings as resulted from the solution of the Security-Constrained OPF based on the Current Distribution Factors. As we can observe, right after the outage two lines are overloaded with line 2-10 reaching about 160% of its limit. We see that with the analytical solution we are able to compute fast a corrective control action for the HVDC line, that will relieve a single overloaded line and will reduce the loading of line 2-10 to about 113%. The solution through the SCOPF results in a somewhat more effective action, but at the cost of significantly higher computation time.

From the short analysis presented above we conclude that the analytical formulation introduced in (30) can provide a fast first reaction for the operator after an outage. The formulation ensures that the loading in the overloaded line will be reduced to a value close to its thermal limit (a deviation should probably be expected due to the approximations). It should be stated here though, that the analytical formulation cannot guarantee that by this action no new overloadings will occur. A simple solution to bypass this shortcoming is to make use of (27). For example, first a corrective control action could be computed through (30) as a first step for a fast reaction. Subsequently, through (27) we can determine if the computed action results to additional overloaded lines. If not, the corrective control action can be implemented by the HVDC. Future work will focus on the development of a simple algorithm in order to ensure that the computed actions would not result to additional overloaded lines.

VII. CONCLUSIONS

In this paper we introduce linear current distribution factors for use in: (a) SCOPF formulations, and (b) in analytical approximations of corrective control actions for HVDC lines. These are relationships based on complex numbers, which can be used in the context of full AC power flow equations.

Through the derivation of these factors, we achieve two goals. First, we extend the range of application of the current injection method by introducing a faster and simpler approach. These factors can be precomputed, similar to the distribution factors in a DC-OPF formulation. Second, these linear relationships allow the incorporation of the HVDC corrective control capabilities in the SCOPF problem.

Three distribution factors are derived: the linear AC outage distribution factor (LOCDF), which determines the line current flows after the occurrence of a line outage; the current distribution factor (CDF), which computes the line currents after a generation outage; and a linear factor which expresses the effect of HVDC corrective control actions on the line currents ($\text{LOCDF}^{\text{PCC}}$).

Two types of case studies are presented in this paper. First, we examine the accuracy of the linear factors. We find that they are almost as accurate as the basic current injection method in [5], [6]. In general, we find that the approximation error of the calculated line flows is within acceptable limits, and, most importantly, it is smaller for higher line loadings. Second, we incorporate the linear factors in an SCOPF problem. We demonstrate the capabilities of the algorithm to take into account generation and line outages and compute corrective control actions of HVDC lines. We compare the optimization results and computation time with the more accurate SCOPF algorithm proposed in [4] in a case study. We find out that the deviation in the optimization results was below 1%, while the algorithm based on the Current Distribution Factors required only 50% of the computation time.

Based on the proposed linear factors, a further contribution of this paper is an approximate analytical solution for corrective control of VSC-HVDC lines. This relationship can be used as a first step for a fast computation of a corrective control action to be taken by the HVDC line. We also present a case study employing the derived relationship.

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